CP-violation in effective field theories

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Wie wundervoll sind diese Wesen,
Die, was nicht deutbar, dennoch deuten,
Was nie geschrieben wurde, lesen,
Verworrenes beherrschend binden
Und Wege noch im Ewig-Dunkeln finden.

Hugo v. Hofmannsthal

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Chapter 1

Introduction

One of the most useful concepts in modern physics is that of symmetries, since this tool allows one to single out fundamental structures from the rich world of physical phenomena. Inseparable from the notion of symmetry are the notions of transformation, invariance and conservation laws. Among the many symmetries, the discrete symmetries like parity P, charge conjugation C and time reversal T play a fundamental role in quantum field theory, since they are the ingredients of the famous CPT-theorem [1], which states that it is impossible to construct a meaningful CPT-noninvariant quantum field theory.

Nevertheless in 1957 it was found [2] that both charge conjugation and parity are drastically violated by the weak interactions, while the product CP was still believed to be conserved. However, in 1964 Christenson, Cronin, Fitch and Turlay [3] discovered CP-violation in the weak two-pion decay of the neutral kaon.

If CP is conserved one can show that the long- and short-lived neutral kaon, K_L and K_S, must form CP eigenstates with quantum numbers $CP = -1$ and $CP = +1$, respectively. Since the two-pion states with zero angular momentum are eigenstates of CP with eigenvalue $+1$, only the short-lived kaon can decay into two pions, while the long-lived kaon decays predominantely into three pions or semileptonically. Thus the decay $K_L \to \pi\pi$ is a definite sign of CP-violation in weak interactions. It can take place if K_L has a small admixture of the CP-odd eigenstate (CP-violation in mixing) or if CP is violated directly in the decay (CP-violation in the amplitudes).

In the fundamental experiment Christenson et al. found that the long-lived kaon decays into two pions with a very small branching ratio. The today values of the branching ratios are given in [4]:

$$\frac{\Gamma(K_L \to \pi^+\pi^-)}{\Gamma_{total}} = (2.03 \pm 0.04) \cdot 10^{-3}\,, \tag{1.1}$$

$$\frac{\Gamma(K_L \to \pi^0\pi^0)}{\Gamma_{total}} = (9.14 \pm 0.34) \cdot 10^{-4}\,. \tag{1.2}$$

CP-violation is mostly measured by the following two ratios of the K_L to the K_S decay rate to two pions:

$$\frac{\Gamma(K_L \to \pi^+\pi^-)}{\Gamma(K_S \to \pi^+\pi^-)} = |\eta_{+-}|^2\,, \tag{1.3}$$

$$\frac{\Gamma(K_L \to \pi^0\pi^0)}{\Gamma(K_S \to \pi^0\pi^0)} = |\eta_{00}|^2\,. \tag{1.4}$$

The today values are given in chapter 3.

The present work is intended to outline some aspects of CP-violation in effective field theories. We will begin with a phenomenological description of CP-violation in the neutral kaon system in chapter 2. Chapter 3 gives a very short introduction to the experimental determination of CP-violating quantities as a completion to chapter 2. In chapter 4 we will study the mechanisms inducing CP-violation by means of simple effective field-theoretical models. Finally, chapter 5 is designated for the investigation of CP-violation in the effective $\Delta S = 1$ nonleptonic weak interaction introduced by Kambor, Missimer and Wyler [5].

Chapter 2

Phenomenology of CP-violation in the neutral kaon system

In this chapter we will outline the basics of the phenomenological description of the neutral kaon system. We will give an overview of a set of phenomenological parameters that allows one to parametrize CP-violation both in mixing and in the amplitudes. The reader is recommended to consult the many reviews on this subject, e.g. [6], [7], [8], for complementary information. Then we will discuss the possibility of absorbing arbitrary phases in the definition of the kaons and its consequences. Finally we will formulate the concept of the mass matrix in field theoretical terms.

2.1 Phenomenology of $K^0 - \overline{K}^0$ mixing

The neutral K-mesons are produced in strong reactions, for example $\pi^- p \to K^0 \Lambda$ or $\pi^+ p \to \overline{K}^0 K^+ p$, and are stable eigenstates of strangeness with eigenvalues ± 1, if the weak interaction is absent. Therefore one always works with these states as far as the strong interactions are concerned. Furthermore K^0 and $\overline{K}^0$ possess a definite third isospin-component $I_3 = \pm 1/2$ and transform as pseudoscalar particles. We choose the phase of the CP-transformation in the following way:

$$CP|K^0\rangle = -|\overline{K}^0\rangle \, , \tag{2.1}$$

$$CP|\overline{K}^0\rangle = -|K^0\rangle \, , \tag{2.2}$$

and we can easily obtain the eigenstates K_1 and K_2 of the CP-operator with eigenvalues $+1$ and -1, respectively:

$$|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K}^0\rangle\right) \quad \to \quad CP|K_1\rangle = +|K_1\rangle \, , \tag{2.3}$$

$$|K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K}^0\rangle\right) \quad \to \quad CP|K_2\rangle = -|K_2\rangle \, . \tag{2.4}$$

In terms of K^0, $\overline{K}^0$-fields we have the relations

$$(CP)K^0(x)(CP)^\dagger = -\overline{K}^0(x_0, -\vec{x}) \, , \tag{2.5}$$

$$(CP)\overline{K}^0(x)(CP)^\dagger = -K^0(x_0, -\vec{x}) \, , \tag{2.6}$$

where the CP-invariance of the vacuum is assumed, and the hermitian combinations

$$K_1 = \frac{i}{\sqrt{2}}(K^0 - \overline{K}^0)\,, \tag{2.7}$$

$$K_2 = \frac{1}{\sqrt{2}}(K^0 + \overline{K}^0) \tag{2.8}$$

transform as

$$(CP)K_1(CP)^\dagger = +K_1\,, \tag{2.9}$$

$$(CP)K_2(CP)^\dagger = -K_2\,. \tag{2.10}$$

However, in the presence of weak interactions, the particles become unstable and experimentally it is found that K^0-decay occurs with two different lifetimes [4]

$$\tau(K_S \to 2\pi) = 0.9 \cdot 10^{-10} sec\,,$$
$$\tau(K_L \to 3\pi) = 0.5 \cdot 10^{-8} sec\,.$$

Thus the K^0's produced by strong interactions seem to be two different particles, K_L and K_S, the long and the short lived kaon, respectively, when we study its weak decays. These states are linear superpositions of the strangeness eigenstates K^0 and $\overline{K}^0$ and they obey the exponential time dependence law

$$|K_L\rangle \to e^{-i\lambda_L\tau}|K_L\rangle \qquad \text{and} \qquad |K_S\rangle \to e^{-i\lambda_S\tau}|K_S\rangle\,, \tag{2.11}$$

where τ is the proper time of the particle. Since the weak interaction does not conserve strangeness, it can induce $K^0 - \overline{K}^0$ transitions and thus we have to consider the $K^0 - \overline{K}^0$-system as a whole. A suitable formalism for studying the decay of a many particle state system is the one evolved by Wigner and Weisskopf [9]. Their concept leads to the result that the decay of a many state system is governed by an effective Schrödinger equation (see appendix A)

$$i\frac{\partial}{\partial t}\psi = \mathcal{M}\psi\,, \tag{2.12}$$

where ψ is an arbitrary state in the $K^0 - \overline{K}^0$-basis and $\mathcal{M} = M - \frac{i}{2}\Gamma$ is the non-hermitian mass matrix. Its hermitian parts are given by

$$M = \frac{1}{2}(\mathcal{M} + \mathcal{M}^\dagger) = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}, \tag{2.13}$$

$$\Gamma = i(\mathcal{M} - \mathcal{M}^\dagger) = \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix}. \tag{2.14}$$

In (2.13), (2.14) and in the following the indices 1, 2 stand for K^0 and $\overline{K}^0$, respectively. From CPT invariance of the weak Hamiltonian, $\mathcal{H}_W$, one can easily derive

$$M_{11} = M_{22} \qquad \text{and} \qquad \Gamma_{11} = \Gamma_{22}\,. \tag{2.15}$$

We assume CPT-invariance in all of our considerations, since all local quantum field theories must obey CPT symmetry [1]. From the hermiticity of M and Γ one has

$$M_{11} = \overline{M_{11}} \qquad \text{and} \qquad \Gamma_{11} = \overline{\Gamma_{11}}\,, \tag{2.16}$$

$$M_{12} = \overline{M_{21}} \qquad \text{and} \qquad \Gamma_{12} = \overline{\Gamma_{21}}\,. \tag{2.17}$$

This leads to the following general form of the mass matrix:

$$\mathcal{M} = \begin{pmatrix} \mathcal{M}_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ \overline{M_{12} - \frac{i}{2}\Gamma_{12}} & \mathcal{M}_{11} \end{pmatrix} . \tag{2.18}$$

The eigenvalues of $\mathcal{M}$ are given by

$$\lambda_{L,S} = M_{L,S} - \frac{i}{2}\Gamma_{L,S} = \mathcal{M}_{11} \pm Q\,, \qquad Q = \sqrt{\mathcal{M}_{12}\cdot\mathcal{M}_{21}}\,, \tag{2.19}$$

where the sign of Q is defined by the condition

$$\Delta\Gamma = \Gamma_L - \Gamma_S = -4\,\mathrm{Im}\,Q < 0 \quad \Rightarrow \quad \mathrm{Im}\,Q > 0\,. \tag{2.20}$$

The real and imaginary part of λ_L and λ_S determine the masses and the decay width of the long and the short lived kaon, respectively. Therefore we obtain the mass difference from

$$\Delta M = M_L - M_S = 2\mathrm{Re}\,Q\,. \tag{2.21}$$

Denoting the K_L - eigenstate of $\mathcal{M}$ in the $K^0 - \overline{K}^0$ -basis with $(1, \tilde{\eta})$ we have

$$|K_L\rangle = \frac{1}{\sqrt{1+|\tilde{\eta}|^2}}\left(|K^0\rangle + \tilde{\eta}|\overline{K}^0\rangle\right)\,, \tag{2.22}$$

$$|K_S\rangle = \frac{1}{\sqrt{1+|\tilde{\eta}|^2}}\left(|K^0\rangle - \tilde{\eta}|\overline{K}^0\rangle\right)\,. \tag{2.23}$$

$\tilde{\eta}$ is the complex number

$$\tilde{\eta} = \frac{Q}{\mathcal{M}_{12}} = \sqrt{\frac{\mathcal{M}_{21}}{\mathcal{M}_{12}}}\,, \tag{2.24}$$

where we have fixed the sign of $\tilde{\eta}$ through (2.20). Writing

$$\rho = \frac{1-\tilde{\eta}}{1+\tilde{\eta}} = \frac{\sqrt{\mathcal{M}_{12}} - \sqrt{\mathcal{M}_{21}}}{\sqrt{\mathcal{M}_{12}} + \sqrt{\mathcal{M}_{21}}} \tag{2.25}$$

we can express the eigenstates in terms of CP eigenstates K_1, K_2:

$$|K_L\rangle = \frac{1}{\sqrt{1+|\rho|^2}}\left(|K_2\rangle + \rho|K_1\rangle\right) \tag{2.26}$$

$$|K_S\rangle = \frac{1}{\sqrt{1+|\rho|^2}}\left(|K_1\rangle + \rho|K_2\rangle\right)\,. \tag{2.27}$$

Note, that we have chosen the arbitrary relative phase of $|K_L\rangle$ and $|K_S\rangle$ in (2.22), (2.23) and (2.26), (2.27) such that

$$\langle K_S|K_L\rangle = \frac{2\mathrm{Re}\,\rho}{\sqrt{1+|\rho|^2}} \geq 0\,. \tag{2.28}$$

ρ is a mixing parameter, which gives the amount of K_1-admixture to K_L, and K_2 to K_S, respectively, i.e. the amount of CP-violation in mixing. This admixture is in fact responsible for CP-violation, since the K_L and K_S states are no longer orthogonal and the K_L can now decay into two pions via the K_1 state.

We can consider now the consequences of the CP-invariance of a theory, in particular its Hamiltonian,

$$(CP)\mathcal{H}(CP)^\dagger = \mathcal{H}\,, \tag{2.29}$$

for the mass matrix. In particular, from (A.27) and (A.28), it follows that the off-diagonal matrix elements are equal,

$$\mathcal{M}_{12} = \mathcal{M}_{21}\,. \tag{2.30}$$

Therefore, M_{12} and Γ_{12} are real according to (2.16), and one has

$$\arg\frac{\Gamma_{12}}{M_{12}} = 0 \bmod \pi \tag{2.31}$$

and thus $\tilde{\eta} = 1$. In addition, from (2.25) and (2.30), we conclude that $\rho = 0$. Therefore, the exponentially decaying states are given by $|K_1\rangle$ and $|K_2\rangle$. A theory with this property is said to have CP-invariance in mixing.

However, a nonzero ρ does not necessarily indicate CP-violation. The reason for this is that we have chosen above a particular phase in the definition of the states $|K^0\rangle$ and $|\overline{K}^0\rangle$. As we will see in section 2.4, a mass matrix where $\tilde{\eta}$ is a pure phase is physically equivalent to the case where the exponentially decaying states are $|K_1\rangle$ and $|K_2\rangle$, whereas one still has (2.31).

Taking into account the freedom in the choice of the phase in the definition of K^0, $\overline{K}^0$, one finds from (2.76) that CP-invariance always leads to (2.31), from where one can easily conclude that

$$\tilde{\eta} = e^{i\phi}, \quad \phi\,\text{real}\,. \tag{2.32}$$

This consideration leads to the following necessary, but not sufficient strangeness phase independent condition for CP-invariance:

$$CP\text{-invariance} \quad \rightarrow \quad \arg\frac{\Gamma_{12}}{M_{12}} = 0\,. \tag{2.33}$$

This is obviously equivalent to the statement:

$$CP\text{-invariance} \quad \rightarrow \quad |\tilde{\eta}| = 1\,. \tag{2.34}$$

However, (2.33) is not a sufficient condition for CP-invariance, but only for CP-invariance in mixing, since the mixing mechanism is not the only source of CP-violation (see figure 2.1). In chapter 4 we will construct a CP-violating model, even though (2.31) is fulfilled.

Having (2.34) or (2.33) at hand we conclude our investigation of indirect CP-violation and turn to the second source of CP-violation, that is CP-violation in the amplitudes.

2.2 The $K \to 2\pi$ amplitudes

In this section we will analyse the decay of the kaons into two pions, $K \to \pi^+\pi^-, \pi^0\pi^0$. Since the kaon is a spin-0 particle, the decay product, in our case two pions, has to be a state with total angular momentum $J = 0$. This means that the two pions are in a symmetric momentum state, and thus their isospin $I = 1$ combines in a symmetric way to a total isospin $I = 0$ or $I = 2$, while the state with $I = 1$ is forbidden by Bose-statistics. We can construct the isospin eigenstates in question, with third isospin component $I_3 = 0$, with the help of the Clebsch-Gordan decomposition:

$$|\pi\pi, I = 0\rangle = \frac{1}{\sqrt{3}} \left(|\pi^+(\vec{k})\pi^-(-\vec{k})\rangle - |\pi^0(\vec{k})\pi^0(-\vec{k})\rangle + |\pi^+(-\vec{k})\pi^-(\vec{k})\rangle \right) , \quad (2.35)$$

$$|\pi\pi, I = 2\rangle = \frac{1}{\sqrt{6}} \left(|\pi^+(\vec{k})\pi^-(-\vec{k})\rangle + 2|\pi^0(\vec{k})\pi^0(-\vec{k})\rangle + |\pi^+(-\vec{k})\pi^-(\vec{k})\rangle \right) . \quad (2.36)$$

The reason for this isospin analysis is that the matrix elements of transitions from K^0 and $\overline{K}^0$ to the same isospin state can be related by Watson's final state theorem. Since the two pions in the final state are scattering only elastically at the energy of the neutral kaon mass via the strong interaction, the theorem in question leads to a $J = 0$, isospin I, $\pi\pi$ phase shift δ_I of the decay amplitudes [8]. Now we can parametrize the amplitudes, i.e. the transition matrix elements of the kaons into the isospin eigenstates in the following way:

$$\langle \pi\pi, I|\mathcal{L}_W(0)|K^0\rangle \doteq iA_I e^{i\delta_I} . \quad (2.37)$$

From CPT-invariance and Watson's final state theorem it follows that

$$\langle \pi\pi, I|\mathcal{L}_W(0)|\overline{K}^0\rangle = -i\overline{A_I} e^{i\delta_I} . \quad (2.38)$$

It is convenient to define the following quantities for later use:

$$\xi_0 = \frac{\text{Im}\, A_0}{\text{Re}\, A_0} \quad \text{and} \quad \xi_2 = \frac{\text{Im}\, A_2}{\text{Re}\, A_2} . \quad (2.39)$$

Direct CP-violation shows up in the lack of relative reality of the amplitudes A_0 and A_2, i.e. in the non-vanishing of $\xi_2 - \xi_0$, as we will discuss in the next section:

$$CP\text{-invariance} \quad \to \quad \arg A_0 = \arg A_2 \bmod \pi . \quad (2.40)$$

However, this is again no sufficient, but a necessary condition for CP-invariance (see figure 2.1). This concludes the discussion of the decay amplitudes $\mathcal{A}(K \to 2\pi)$ and we turn now to the parametrization of CP-violation in neutral kaon decays.

2.3 CP-violating parameters

As mentioned in the introduction the CP-violation signal is provided by the asymmetries

$$\eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} , \quad (2.41)$$

$$\eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} . \quad (2.42)$$

In order to calculate these two ratios it is helpful to parametrize them in terms of some other physical quantities. One possible quantity we can define to characterize the amount of CP-violation in $K \to 2\pi$ transitions is the ε parameter:

$$\varepsilon \doteq \frac{A(K_L \to 2\pi, I=0)}{A(K_S \to 2\pi, I=0)} . \tag{2.43}$$

It is physical in the sense that it is phase convention independent (see section 2.4). However, it is not directly accessible to experiment, since we are dealing with pure isospin states. We can express this parameter in terms of ρ and ξ_0 with the help of (2.26), (2.27) and (2.39) in the following way

$$\varepsilon = \frac{\rho + i\xi_0}{1 + i\rho\xi_0} . \tag{2.44}$$

Two other physical, i.e. phase convention independent ratios are

$$\frac{A(K_L \to 2\pi, I=2)}{A(K_S \to 2\pi, I=0)} \qquad \text{and} \qquad \omega \doteq \frac{A(K_S \to 2\pi, I=2)}{A(K_S \to 2\pi, I=0)} , \tag{2.45}$$

which again can be expressed in terms of the mixing parameter ρ and the complex isospin amplitudes A_I:

$$\frac{A(K_L \to 2\pi, I=2)}{A(K_S \to 2\pi, I=0)} = \frac{i\frac{\text{Im } A_2}{\text{Re } A_0} + \rho\frac{\text{Re } A_2}{\text{Re } A_0}}{1 + i\rho\xi_0} e^{i(\delta_2 - \delta_0)} \tag{2.46}$$

and

$$\omega = \frac{A(K_S \to 2\pi, I=2)}{A(K_S \to 2\pi, I=0)} = \frac{\frac{\text{Re } A_2}{\text{Re } A_0} + i\rho\frac{\text{Im } A_2}{\text{Re } A_0}}{1 + i\rho\xi_0} e^{i(\delta_2 - \delta_0)} . \tag{2.47}$$

Note, that ω is not a parameter indicating CP-violation, but measures the fraction of the $\Delta I = \frac{3}{2}$- to the $\Delta I = \frac{1}{2}$-transitions. It is therefore a quantity showing the deviation from the $\Delta I = \frac{1}{2}$-rule. Now we define the ε'-parameter as the following combination of these natural ratios

$$\varepsilon' \doteq \frac{1}{\sqrt{2}} \left(\frac{A(K_L \to 2\pi, I=2)}{A(K_S \to 2\pi, I=0)} - \varepsilon \cdot \omega \right) . \tag{2.48}$$

After some rearrangements we finally obtain

$$\begin{aligned} \varepsilon' &= \frac{i}{\sqrt{2}} \frac{(1-\rho^2) e^{i(\delta_2 - \delta_0)}}{(1 + i\rho\xi_0)^2} \frac{1}{(\text{Re}A_0)^2} (\text{Im}A_2 \text{Re}A_0 - \text{Im}A_0 \text{Re}A_2) \\ &= \frac{i}{\sqrt{2}} \frac{(1-\rho^2) e^{i(\delta_2 - \delta_0)}}{(1 + i\rho\xi_0)^2} \frac{\text{Re}A_2}{\text{Re}A_0} (\xi_2 - \xi_0) , \end{aligned} \tag{2.49}$$

which shows clearly the fact of ε' being a parameter measuring the lack of relative reality between the two isospin amplitudes A_0 and A_2. This parameter accounts thus only for intrinsic CP-violation specific to the the $K \to 2\pi$ decay, in contrast to the CP-violation in mixing.

We are now able to express the experimentally most important CP-violating parameters η_{00} and η_{+-} in terms of ε, ε' and ω:

$$\eta_{+-} = \varepsilon + \frac{\varepsilon'}{1 + \frac{1}{\sqrt{2}}\omega} , \tag{2.50}$$

$$\eta_{00} = \varepsilon - \frac{2\varepsilon'}{1 - \sqrt{2}\omega} . \tag{2.51}$$

Up to now we have made no approximation in the derivation of the above expressions. However, it is useful to thin down the exact expressions by neglecting terms which are quadratic in the CP-violating parameters, since these parameters are experimentally known to be very small. We then obtain the following set of expressions for the CP-violating parameters

$$\omega \simeq \frac{\mathrm{Re}A_2}{\mathrm{Re}A_0} e^{i(\delta_2-\delta_0)}\,, \tag{2.52}$$

$$\varepsilon \simeq \rho + i\xi_0\,, \tag{2.53}$$

$$\varepsilon' \simeq \frac{i}{\sqrt{2}}\frac{\mathrm{Re}A_2}{\mathrm{Re}A_0}(\xi_2-\xi_0)e^{i(\delta_2-\delta_0)}\,, \tag{2.54}$$

while the expressions for η_{00} and η_{+-} remain the same.

2.4 Strangeness transformation

In this section we will discuss the possibility of absorbing arbitrary phases in the definition of the kaons, and its consequences for the CP-violating parameters. Since the kaon states have non-zero strangeness and the K^0, $\overline{K}^0$ are defined only by strangeness-conserving strong interactions, one can redefine the states by using a strangeness transformation,

$$|K^0\rangle_\alpha = e^{-i\alpha S}|K^0\rangle = e^{-i\alpha}|K^0\rangle\,, \tag{2.55}$$

$$|\overline{K}^0\rangle_\alpha = e^{-i\alpha S}|\overline{K}^0\rangle = e^{i\alpha}|\overline{K}^0\rangle\,, \tag{2.56}$$

where S is the strangeness operator:

$$S|K^0\rangle = +|K^0\rangle \quad \text{and} \quad S|\overline{K}^0\rangle = -|\overline{K}^0\rangle\,. \tag{2.57}$$

We define the CP-transformation in this new basis as before:

$$(CP)_\alpha|K^0\rangle_\alpha \doteq -|\overline{K}^0\rangle_\alpha \quad \text{or} \quad (CP)_\alpha \doteq e^{-i\alpha S}(CP)e^{i\alpha S}\,, \tag{2.58}$$

which leaves the CP-even and CP-odd states unchanged:

$$|K_1\rangle_\alpha = \frac{1}{\sqrt{2}}(|K^0\rangle_\alpha - |\overline{K}^0\rangle_\alpha) \quad \to \quad (CP)_\alpha|K_1\rangle_\alpha = +|K_1\rangle_\alpha\,, \tag{2.59}$$

$$|K_2\rangle_\alpha = \frac{1}{\sqrt{2}}(|K^0\rangle_\alpha + |\overline{K}^0\rangle_\alpha) \quad \to \quad (CP)_\alpha|K_2\rangle_\alpha = -|K_2\rangle_\alpha\,. \tag{2.60}$$

In terms of the fields K^0, $\overline{K}^0$ we have

$$[S, K^0] = -K^0 \quad \text{and} \quad [S, \overline{K}^0] = +\overline{K}^0\,. \tag{2.61}$$

Using the identity

$$e^{-A}Be^{A} = B - [A,B] + \frac{1}{2!}[A,[A,B]] - \ldots\,, \tag{2.62}$$

we get

$$K^0_\alpha = e^{-i\alpha S}K^0e^{+i\alpha S} = e^{+i\alpha}K^0\,, \tag{2.63}$$

$$\overline{K}^0_\alpha = e^{-i\alpha S}\overline{K}^0e^{+i\alpha S} = e^{-i\alpha}\overline{K}^0\,. \tag{2.64}$$

The action of the CP-transformation is again defined as before:

$$(CP)_\alpha K^0_\alpha (CP)^\dagger_\alpha \doteq -\overline{K}^0_\alpha \,, \tag{2.65}$$
$$(CP)_\alpha \overline{K}^0_\alpha (CP)^\dagger_\alpha \doteq -K^0_\alpha \,, \tag{2.66}$$

with $(CP)_\alpha \doteq e^{-i\alpha S}(CP)e^{i\alpha S}$. Thus $K_1 = \frac{i}{\sqrt{2}}(K^0_\alpha - \overline{K}^0_\alpha)$ and $K_2 = \frac{1}{\sqrt{2}}(K^0_\alpha + \overline{K}^0_\alpha)$ are still the CP-even and CP-odd combination, respectively:

$$(CP)_\alpha \frac{i}{\sqrt{2}}(K^0_\alpha - \overline{K}^0_\alpha)(CP)^\dagger_\alpha = +\frac{i}{\sqrt{2}}(K^0_\alpha - \overline{K}^0_\alpha) \tag{2.67}$$
$$(CP)_\alpha \frac{1}{\sqrt{2}}(K^0_\alpha + \overline{K}^0_\alpha)(CP)^\dagger_\alpha = -\frac{1}{\sqrt{2}}(K^0_\alpha + \overline{K}^0_\alpha)\,. \tag{2.68}$$

Therefore it is possible to transform the CP-odd combination K_2 into the CP-even combination K_1, and vice versa, if we use a strangeness transformation with $\alpha = -\frac{\pi}{2}$:

$$e^{-i\frac{\pi}{2}S}K_2 e^{+i\frac{\pi}{2}S} = \frac{1}{\sqrt{2}}(e^{+i\frac{\pi}{2}}K^0_{-\frac{\pi}{2}} + e^{-i\frac{\pi}{2}}\overline{K}^0_{-\frac{\pi}{2}}) = \frac{i}{\sqrt{2}}(K^0_{-\frac{\pi}{2}} - \overline{K}^0_{-\frac{\pi}{2}}) = (K_1)_{-\frac{\pi}{2}}\,. \tag{2.69}$$

The same effect can be achieved by a redefinition of the CP-transformation in (2.1) and (2.2) as well, because we have

$$(CP)_\alpha K^0 (CP)^\dagger_\alpha = -e^{-i2\alpha}\overline{K}^0 \,, \tag{2.70}$$
$$(CP)_\alpha \overline{K}^0 (CP)^\dagger_\alpha = -e^{+i2\alpha}K^0 \,. \tag{2.71}$$

Choosing again $\alpha = -\frac{\pi}{2}$ we get

$$(CP)_\alpha K^0 (CP)^\dagger_\alpha = +\overline{K}^0 \,, \tag{2.72}$$
$$(CP)_\alpha \overline{K}^0 (CP)^\dagger_\alpha = +K^0 \,, \tag{2.73}$$

and thus, in contrast to (2.9) and (2.10),

$$(CP)_\alpha K_1 (CP)^\dagger_\alpha = -K_1 \,, \tag{2.74}$$
$$(CP)_\alpha \overline{K}_2 (CP)^\dagger_\alpha = +K_2 \,. \tag{2.75}$$

Therefore, CP-invariance of a theory means that there exists a phase α such that the theory is invariant under a $(CP)_\alpha$-transformation.

However, the freedom in redefining K^0, $\overline{K}^0$ must have no effects on physical parameters, since any measurable quantity must be phase convention independent. By looking at the transformation properties of the parameters involved in the description of CP-violation, we find:

$$M^\alpha_{12} = e^{2i\alpha}M_{12}\,, \qquad \Gamma^\alpha_{12} = e^{2i\alpha}\Gamma_{12}\,, \tag{2.76}$$
$$\tilde{\eta}_\alpha = e^{-2i\alpha}\tilde{\eta}\,, \tag{2.77}$$
$$\rho_\alpha = \frac{-1+\rho+e^{2i\alpha}(1+\rho)}{1-\rho+e^{2i\alpha}(1+\rho)}\,, \tag{2.78}$$
$$A^\alpha_I = e^{-i\alpha}A_I\,, \tag{2.79}$$

while M_{11}, Γ_{11} and Q remain invariant, since they lead to ΔM and $\Gamma_L - \Gamma_S$ which are both measurable quantities (see (2.20) and (2.21)). The action of a strangeness transformation on the physical states $|K_L\rangle$ and $|K_S\rangle$ is given by

$$|K_{L,S}\rangle_\alpha = e^{i\alpha'}|K_{L,S}\rangle \,, \tag{2.80}$$

since

$$\frac{1+\rho_\alpha}{\sqrt{1+|\rho_\alpha|^2}}e^{-i\alpha} = \frac{1+\rho}{\sqrt{1+|\rho|^2}}e^{i\alpha'} \,, \tag{2.81}$$

$$\frac{-1+\rho_\alpha}{\sqrt{1+|\rho_\alpha|^2}}e^{i\alpha} = \frac{-1+\rho}{\sqrt{1+|\rho|^2}}e^{i\alpha'} \,, \tag{2.82}$$

where α' depends on ρ and α in a non-trivial way. This means in particular that the quantities η_{+-}, η_{00}, ε, ε' and ω as defined in (2.41), (2.42), (2.43), (2.47) and (2.48), respectively, are phase convention independent, since they are ratios of transitions of $K_{L,S}$ to 2π.

In the literature one often works with the Wu-Yang phase convention [10], where the phases of K^0, $\overline{K}^0$ are chosen such that the imaginary part of A_0, and thus ξ_0, vanishes. In this phase convention the CP-violating quantity ε is directly related to the mixing parameter: $\varepsilon = \rho$. However, as already stated, a non-vanishing ρ does not necessarily mean CP-violation in mixing. Thus one has to check thouroghly which phase convention is used in the description in question, in order to prevent confusion.

In the previous section we have already stressed the importance of phase convention independent formulations of CP-violation. From (2.76) and (2.77) it can be seen that the statements (2.33) and (2.34), respectively, are in fact independent of a strangeness phase transformation, as well as (2.40). As a summary we can make the following statement (see also figure 2.1):

$$CP\text{-invariance} \quad \to \quad 2\arg A_0 = 2\arg A_2 = \arg\Gamma_{12} \bmod \pi \,. \tag{2.83}$$

We will discuss this formulation of CP-invariance by means of field theoretical models in chapter 4.

2.5 The mass matrix in a field theoretical formulation

In this section we will make the connection between the mass matrix $\mathcal{M}$, formulated by Wigner and Weisskopf [9] in the quantum mechanical language (see appendix A), and the two point function, in particular the 1PI-function, which is a field theoretical concept. This is in order to discuss CP-violation in field theoretical models in terms of the two point function, i.e. the self energy.

We begin by considering the two-point function of two kaon fields,

$$\frac{1}{i}\Delta(x) = \langle 0|T\left(K^0(x)\overline{K}^0(0)\right)|0\rangle \,, \tag{2.84}$$

and write its Fourier-transform as

$$\Delta(p^2) = \left[M^2 - p^2 - \Sigma(p^2)\right]^{-1} \,, \tag{2.85}$$

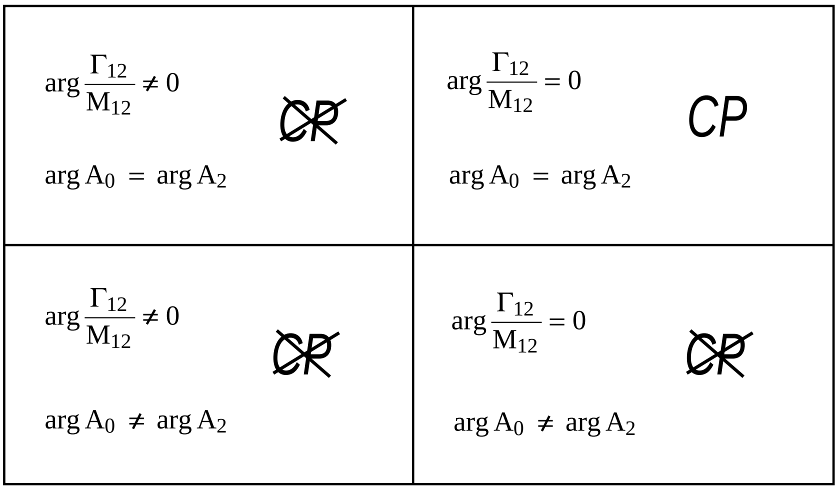

Figure 2.1: A graphical representation of how CP-violation or -invariance may manifest.

where M is the mass of the free particle. The self energy Σ is in general a complex quantity. The real part of the denominator has a zero at

$$M_{ph.}^2 = M^2 - \mathrm{Re}\,\Sigma(M_{ph.}^2)\,. \tag{2.86}$$

We interprete in the following $M_{ph.}$ as the physical mass of the kaon.

In this work we concentrate on field-theoretical models where Σ is obtained through a perturbative expansion in a Lagrangian framework where

$$\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{int}\,. \tag{2.87}$$

In this case, Σ amounts to the 1PI contributions of the two-point function, i.e. graphs that remain connected if a kaon line is cut. In the following, we consider interaction Lagrangians of the form

$$\mathcal{L}_{int} = \epsilon_1 \mathcal{L}_{int}^{(1)} + \epsilon_2 \mathcal{L}_{int}^{(2)}\,, \tag{2.88}$$

where ϵ_i are small parameters, with $\epsilon_1 = \mathcal{O}(\epsilon_2^2)$, and where $\mathcal{L}_{int}^{(1)}$ contains terms quadratic in the fields (e.g. counterterms). To order ϵ_2^2, one then has

$$M_{ph.}^2 = M^2 - \mathrm{Re}\,\Sigma(M^2)\,, \tag{2.89}$$

and

$$\Sigma(M^2) = \langle p|\mathcal{L}_{int}(0)|p\rangle + \frac{i}{2}\int d^4x \langle p|T\left(\mathcal{L}_{int}(x)\mathcal{L}_{int}(0)\right)|p\rangle_{1\mathrm{PI}}\,. \tag{2.90}$$

Since the diagonal element $\mathcal{M}_{11}$ of the kaon mass matrix is the (complex) mass with which the particle propagates, one has furthermore in this approximation

$$\mathcal{M}_{11} = M\sqrt{1 - \frac{\Sigma}{M^2}} = M - \frac{1}{2M}\Sigma\,. \tag{2.91}$$

For the non-diagonal elements, we use

$$\mathcal{M}_{12} = -\frac{\Sigma_{12}}{2M}\,, \tag{2.92}$$

where

$$\Sigma_{12} = \langle p,1|\mathcal{L}_{int}(0)|p,2\rangle + \frac{i}{2}\int d^4x \langle p,1|T\left(\mathcal{L}_{int}(x)\mathcal{L}_{int}(0)\right)|p,2\rangle_{\rm 1PI}\,, \tag{2.93}$$

where the indices 1, 2 denote the K^0 and $\overline{K}^0$, respectively.

Chapter 3

Experiments of CP-violation in neutral kaon decays

3.1 Determination of η_{+-} and η_{00}

As already mentioned, the most important quantities describing CP-violation in neutral kaon decays are the two ratios

$$\eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} \doteq |\eta_{+-}|e^{i\phi_{+-}}\,, \tag{3.1}$$

$$\eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} \doteq |\eta_{00}|e^{i\phi_{00}}\,, \tag{3.2}$$

since their phases and amplitudes are both directly accessible by experiment. They can be determined by looking at the intensity of $\pi\pi$ decays in a neutral kaon beam as a function of proper time τ.

The beam of kaons is produced by sending high-energy protons into a target (e.g. beryllium [11]), where mostly K^0 together with a smaller number of $\overline{K}^0$ emerge. At $\tau = 0$ the beam may be represented by the state

$$|\psi\rangle = N_K|K^0\rangle + N_{\overline{K}}|\overline{K}^0\rangle\,, \tag{3.3}$$

where N_K and $N_{\overline{K}}$ give the fraction of the produced particles. Since the kaons are incoherent, the beam evolves in time in a complicated manner,

$$\begin{aligned} |\psi(\tau)\rangle = {} & \tilde{N}_K \frac{1}{1+\rho}\left\{e^{-i\lambda_L\tau}|K_L\rangle + e^{-i\lambda_S\tau}|K_S\rangle\right\} \\ & + \tilde{N}_{\overline{K}}\frac{1}{1-\rho}\left\{e^{-i\lambda_L\tau}|K_L\rangle - e^{-i\lambda_S\tau}|K_S\rangle\right\}\,. \end{aligned} \tag{3.4}$$

Squaring the transition amplitude $\langle\pi\pi|T|\psi(\tau)\rangle$ we get the intensity of the $\pi\pi$ decays, $I(\psi(\tau) \to \pi\pi)$. After some calculation one obtains the following expression (for simplicity we only give the decay intensity for a pure K^0- and $\overline{K}^0$-beam, respectively):

$$I(K^0(\tau), \overline{K}^0(\tau) \to \pi\pi) \sim e^{-\Gamma_S\tau} + |\eta|^2 e^{-\Gamma_L\tau} \pm 2|\eta|e^{-\frac{1}{2}(\Gamma_L+\Gamma_S)\tau}\cos(\Delta M\tau - \phi)\,, \tag{3.5}$$

where η, ϕ means η_{+-}, ϕ_{+-} or η_{00}, ϕ_{00}, respectively. Since the mass difference of the long- and short-lived kaon, $\Delta M = M_L - M_S$, and their decay rates, Γ_L and Γ_S, can be measured independently by different experiments, (3.5) allows one to determine both the phase and the amplitude of η_{+-} and η_{00}, respectively.

The measured values are given in [4]:

$$
\begin{aligned}
|\eta_{00}| &= (2.259 \pm 0.023) \cdot 10^{-3}\,, && (3.6)\\
|\eta_{+-}| &= (2.269 \pm 0.023) \cdot 10^{-3}\,, && (3.7)\\
\phi_{00} &= 43.3^\circ \pm 1.3^\circ\,, && (3.8)\\
\phi_{+-} &= 44.3^\circ \pm 0.8^\circ\,. && (3.9)
\end{aligned}
$$

3.2 Determination of $\frac{\varepsilon'}{\varepsilon}$

We have seen in the previous sections that ε' measures CP-violation in the amplitudes. Therefore, a deviation of the ratio $\frac{\varepsilon'}{\varepsilon}$ from zero is a sign of direct CP-violation in neutral kaon decays. The first evidence for direct CP-violation has been given in 1988 by Burkhardt et al. [12].

Now $\frac{\varepsilon'}{\varepsilon}$ can be determined from the double ratio of K_S and K_L decay rates into charged and neutral pions,

$$
\left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 = \frac{\Gamma(K_L \to 2\pi^0)/\Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K_S \to 2\pi^0)/\Gamma(K_L \to \pi^+\pi^-)}\,, \tag{3.10}
$$

if we use (2.41) and (2.42) and neglect all terms quadratic in small quantities (e.g. $\omega\varepsilon'$):

$$
\begin{aligned}
\eta_{+-} &\simeq \varepsilon + \varepsilon'\,, && (3.11)\\
\eta_{00} &\simeq \varepsilon - 2\varepsilon'\,. && (3.12)
\end{aligned}
$$

We obtain

$$
\mathrm{Re}\,\frac{\varepsilon'}{\varepsilon} \simeq \frac{1}{6}\left(1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2\right) \simeq \frac{1}{3}\left(1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|\right)\,. \tag{3.13}
$$

The central point in the derivation of (3.13) is that we have to a good approximation:

$$
\frac{\varepsilon'}{\varepsilon} \simeq \mathrm{Re}\,\frac{\varepsilon'}{\varepsilon} \simeq \left|\frac{\varepsilon'}{\varepsilon}\right|\,, \tag{3.14}
$$

since the phases of ε' and ε are experimentally known to be very close. To be more precise the phase of ε' is determined by (2.48) and the measured value of the $\pi\pi$ phase shift, while the phase of ε is given by [4]

$$
\phi(\varepsilon) \approx \arctan\left(2\frac{\Delta M}{\Gamma_S}\right)\,. \tag{3.15}
$$

In the experiment one measures the double ratio of the four modes in (3.10) simultaneously. This allows the elimination of common systematic errors and the determination of $\frac{\varepsilon'}{\varepsilon}$ to a very high precision. The measured values can be found in [4]:

$$
\begin{aligned}
\left|\frac{\eta_{00}}{\eta_{+-}}\right| &= 0.9955 \pm 0.0023\,, && (3.16)\\
\frac{\varepsilon'}{\varepsilon} &= (1.5 \pm 0.8) \cdot 10^{-3}\,. && (3.17)
\end{aligned}
$$

Chapter 4

Effective CP-violating models

In this chapter we discuss CP-violation in the framework of quantum field theory of scalar fields. Since CPT-symmetry is guaranteed in this context [1], and since T-transformation are antiunitary and thus involve complex conjugation, one expects that CP-violation is induced by coupling constants having a nonzero imaginary part.

We investigate several models of increasing complexity that allow us to calculate the CP-violating parameters η_{+-} and η_{00}, using ordinary perturbation theory (loop expansion).

4.1 A model with CP-invariance in mixing and $\varepsilon' \neq 0$

We begin by considering a simple model describing the decay of neutral kaons into two pions,

$$\begin{aligned} \mathcal{L}_1 &= \mathcal{L}_{free} + \mathcal{L}_1' \,, \\ \mathcal{L}_1' &= \beta K^0\pi^0\pi^0 + \gamma K^0\pi^+\pi^- + h.c. \,, \end{aligned} \tag{4.1}$$

where β and γ are arbitrary complex coupling constants. In order to calculate the CP-violating parameters η_{+-} and η_{00} we first determine the exponentially decaying states $K_{1,2}$ via the mass matrix.

4.1.1 The mass matrix and CP-invariance in mixing

As outlined in section 2.5 we calculate the elements of the mass matrix via the self energy of the kaon. $\mathcal{L}_1'$ contributes with neutral and charged pion loops in second order. We obtain:

$$\Sigma_{11}(p^2) = \Sigma_{22}(p^2) = (2\beta\overline{\beta} + \gamma\overline{\gamma})\overline{J}(p^2; M_\pi, M_\pi) \,, \tag{4.2}$$

$$\Sigma_{12}(p^2) = (2\overline{\beta}^2 + \overline{\gamma}^2)\overline{J}(p^2; M_\pi, M_\pi) \,, \tag{4.3}$$

$$\Sigma_{21}(p^2) = (2\beta^2 + \gamma^2)\overline{J}(p^2; M_\pi, M_\pi) \,, \tag{4.4}$$

where $\overline{J}(p^2; M_\pi, M_\pi)$ is defined through

$$J\left(p^2; M_\pi, M_\pi\right) = \frac{1}{i}\int \frac{d^d l}{(2\pi)^d}\frac{1}{l^2 - {M_\pi}^2}\cdot\frac{1}{(p-l)^2 - {M_\pi}^2} \tag{4.5}$$

$$= J\left(0; M_\pi, M_\pi\right) + \overline{J}\left(p^2; M_\pi, M_\pi\right) \,, \tag{4.6}$$

and where we have absorbed the divergent parts proportional to $J\,(0; M_\pi, M_\pi)$ in the counterterm Lagrangian

$$\mathcal{L}_{ct} = C_1 K^0 \overline{K}^0 + C_2 K^0 K^0 + \overline{C_2}\,\overline{K}^0 \overline{K}^0 \tag{4.7}$$

with suitably chosen couplings C_i.

Thus we have for the mass matrix elements

$$\begin{aligned}
\mathcal{M}_{11} = \mathcal{M}_{22} &= M_K - (2\beta\overline{\beta} + \gamma\overline{\gamma})\frac{1}{2M_K}\overline{J}(M_K^2; M_\pi, M_\pi)\,, && (4.8)\\
\mathcal{M}_{12} &= -(2\overline{\beta}^2 + \overline{\gamma}^2)\frac{1}{2M_K}\overline{J}(M_K^2; M_\pi, M_\pi)\,, && (4.9)\\
\mathcal{M}_{21} &= -(2\beta^2 + \gamma^2)\frac{1}{2M_K}\overline{J}(M_K^2; M_\pi, M_\pi)\,. && (4.10)
\end{aligned}$$

We can decompose the mass matrix in its hermitian parts,

$$M = \frac{1}{2}(\mathcal{M} + \mathcal{M}^\dagger) \quad \text{and} \quad \Gamma = i(\mathcal{M} - \mathcal{M}^\dagger)\,, \tag{4.11}$$

which yields

$$\begin{aligned}
M_{11} = M_{22} &= M_K - (2\beta\overline{\beta} + \gamma\overline{\gamma})\frac{1}{2M_K}\mathrm{Re}\,\overline{J}(M_K^2; M_\pi, M_\pi)\,, && (4.12)\\
M_{12} = \overline{M_{21}} &= -(2\overline{\beta}^2 + \overline{\gamma}^2)\frac{1}{2M_K}\mathrm{Re}\,\overline{J}(M_K^2; M_\pi, M_\pi)\,, && (4.13)\\
\Gamma_{11} = \Gamma_{22} &= (2\beta\overline{\beta} + \gamma\overline{\gamma})\frac{1}{M_K}\mathrm{Im}\,\overline{J}(M_K^2; M_\pi, M_\pi)\,, && (4.14)\\
\Gamma_{12} = \overline{\Gamma_{21}} &= (2\overline{\beta}^2 + \overline{\gamma}^2)\frac{1}{M_K}\mathrm{Im}\,\overline{J}(M_K^2; M_\pi, M_\pi)\,. && (4.15)
\end{aligned}$$

The eigenvalues of $\mathcal{M}$ are given by

$$\lambda_{L,S} = M_{L,S} - \frac{i}{2}\Gamma_{L,S} = \mathcal{M}_{11} \pm Q\,, \qquad Q = \sqrt{\mathcal{M}_{12}\cdot\mathcal{M}_{21}}\,, \tag{4.16}$$

where the sign of Q is defined by the condition

$$\Delta\Gamma = \Gamma_L - \Gamma_S = -4\,\mathrm{Im}Q < 0 \quad \Rightarrow \quad \mathrm{Im}Q > 0\,. \tag{4.17}$$

Denoting the K_L - eigenstate of $\mathcal{M}$ in the $K^0 - \overline{K}^0$ -basis with $(1, \tilde{\eta})$ we have

$$\tilde{\eta} = \sqrt{\frac{\mathcal{M}_{21}}{\mathcal{M}_{12}}} = \frac{Q}{\mathcal{M}_{12}} = -\frac{(2\beta^2 + \gamma^2)}{|2\beta^2 + \gamma^2|} = e^{i\varphi_{\tilde{\eta}}} \tag{4.18}$$

showing the fact that $\tilde{\eta}$ is a pure phase in the one-loop approximation. This phase can be transformed away with a redefinition of the kaon states with the help of a strangeness transformation (see section 2.4), since $\tilde{\eta}$ transforms then as

$$\tilde{\eta} \to e^{-i2\alpha}\tilde{\eta}\,, \tag{4.19}$$

where α is the strangeness transformation angle. Choosing $\alpha = \varphi_{\tilde{\eta}}/2$ we get $\tilde{\eta} = 1$ and thus $\rho = 0$ which leads to $K_L \sim K_2$ and $K_S \sim K_1$, respectively, indicating CP-invariance in mixing.

4.1.2 The $K \to 2\pi$-amplitudes and direct CP-violation

Let us now calculate the transition amplitudes for the $K \to 2\pi$-decay in order to find the CP-violating quantities η_{0+-} and η_{00}. Our model interaction Lagrangian (4.1) induces both $K \to \pi^0\pi^0$ and $K \to \pi^+\pi^-$ transitions:

$$\langle \pi^0\pi^0 | \mathcal{L}_1'(0) | K^0 \rangle = -2\beta \,, \tag{4.20}$$

$$\langle \pi^+\pi^- | \mathcal{L}_1'(0) | K^0 \rangle = \gamma \,. \tag{4.21}$$

Using the Clebsch-Gordan decomposition into states with definite isospin, and taking into account that we have no $\pi\pi$ final state interactions and thus $\delta_1 = \delta_2 = 0$, we obtain

$$\langle \pi\pi, I=0 | \mathcal{L}_1'(0) | K^0 \rangle = \frac{2}{\sqrt{3}}(\gamma + \beta) \doteq iA_0 \,, \tag{4.22}$$

$$\langle \pi\pi, I=2 | \mathcal{L}_1'(0) | K^0 \rangle = \sqrt{\frac{2}{3}}(\gamma - 2\beta) \doteq iA_2 \,, \tag{4.23}$$

or

$$A_0 = \frac{2}{\sqrt{3}}(\gamma_2 + \beta_2 - i(\gamma_1 + \beta_1)) \,, \tag{4.24}$$

$$A_2 = \sqrt{\frac{2}{3}}(\gamma_2 - 2\beta_2 - i(\gamma_1 - 2\beta_1)) \,, \tag{4.25}$$

where $\beta = \beta_1 + i\beta_2$ and $\gamma = \gamma_1 + i\gamma_2$, β_i, γ_i real. We are now able to calculate ω and the parameters ε, ε' that measures CP-violation:

$$\omega = \frac{1}{\sqrt{2}} \frac{\gamma_2 - 2\beta_2 - (\gamma_1 - 2\beta_1)\tan\frac{\varphi_{\tilde{\eta}}}{2}}{\gamma_2 + \beta_2 - (\gamma_1 + \beta_1)\tan\frac{\varphi_{\tilde{\eta}}}{2}} \,, \tag{4.26}$$

$$\varepsilon = -i \tan\left(\frac{\varphi_{\tilde{\eta}}}{2} - \arg A_0\right) \,, \tag{4.27}$$

$$\varepsilon' = \frac{3i}{2}(\beta_1\gamma_2 - \beta_2\gamma_1) \cdot \frac{1 + \tan^2\frac{\varphi_{\tilde{\eta}}}{2}}{(\beta_2 + \gamma_2 - (\gamma_1 + \beta_1)\tan\frac{\varphi_{\tilde{\eta}}}{2})^2} \,, \tag{4.28}$$

where $\varphi_{\tilde{\eta}} = \arg(2\beta^2 + \gamma^2) \pm \pi$. The calculation of η_{+-} and η_{00} is now straightforward:

$$\eta_{+-} = -i \tan\left(\frac{\varphi_{\tilde{\eta}}}{2} - \arg A_0\right) + i \frac{(1 + \tan^2\frac{\varphi_{\tilde{\eta}}}{2})(\beta_1\gamma_2 - \beta_2\gamma_1)}{(\gamma_2 + \beta_2 - (\gamma_1 + \beta_1)\tan\frac{\varphi_{\tilde{\eta}}}{2})\,(\gamma_2 - \gamma_1 \tan\frac{\varphi_{\tilde{\eta}}}{2})} \,, \tag{4.29}$$

$$\eta_{00} = -i \tan\left(\frac{\varphi_{\tilde{\eta}}}{2} - \arg A_0\right) - i \frac{(1 + \tan^2\frac{\varphi_{\tilde{\eta}}}{2})(\beta_1\gamma_2 - \beta_2\gamma_1)}{(\gamma_2 + \beta_2 - (\gamma_1 + \beta_1)\tan\frac{\varphi_{\tilde{\eta}}}{2})\,(\beta_2 - \beta_1 \tan\frac{\varphi_{\tilde{\eta}}}{2})} \,. \tag{4.30}$$

All these quantities show that (4.1) is in general a model inducing direct CP-violation, i.e. $\varepsilon' \neq 0$, but no CP-violation in mixing, i.e. the exponentially decaying states are $|K_1\rangle$ and $|K_2\rangle$.

However, if $\beta_1\gamma_2 = \beta_2\gamma_1$, or in other words if the phases φ_β and φ_γ of the two couplings β and γ fulfill $\varphi_\beta = \varphi_\gamma \bmod \pi$, we have no CP-violation at all, and we can apply a strangeness transformation, e.g. with $\alpha = \varphi_{\tilde\eta}/2 = \varphi_\beta - \frac{\pi}{2}$, in order to redefine the phases of K^0, $\overline{K}^0$ in $\mathcal{L}_1'$ such that the CP-invariance is explicit in the interaction Lagrangian:

$$\begin{aligned}
\mathcal{L}_1' &= |\beta|(e^{i(\varphi_\beta-\alpha)}K^0_\alpha + e^{-i(\varphi_\beta-\alpha)}\overline{K}^0_\alpha)\pi^0\pi^0 \\
&\quad +|\gamma|(e^{i(\varphi_\beta-\alpha)}K^0_\alpha + e^{-i(\varphi_\beta-\alpha)}\overline{K}^0_\alpha)\pi^+\pi^- \qquad (4.31)\\
&= i|\beta|(K^0_\alpha - \overline{K}^0_\alpha)\pi^0\pi^0 + i|\gamma|(K^0_\alpha - \overline{K}^0_\alpha)\pi^+\pi^- \,, \qquad (4.32)
\end{aligned}$$

where we assumed that $\varphi_\beta = \varphi_\gamma$. Furthermore, the CP-invariance shows up in ε, ε', η_{00} and η_{+-} all becoming zero. In particular, ε vanishes because $2\arg A_0 = 2\arg A_2 = \arg\tilde\eta$ (see 2.83).

We therefore conclude that in this model the non-vanishing phase difference of the couplings that induce transitions of the kaon to $2\pi^0$ and $\pi^+\pi^-$, respectively, is in fact a necessary condition for direct CP-violation.

We now turn to the analysis of mechanisms that induce CP-violation in mixing.

4.2 A model with CP-violation in mixing and $\varepsilon' \neq 0$

Our starting point is an interaction described by the Lagrangian (4.1), where $\varphi_\beta \neq \varphi_\gamma \bmod \pi$. As outlined in the previous section this phase difference of the couplings guarantees direct CP-violation, i.e. $\varepsilon' \neq 0$. Now CP-violation in mixing can be realised in our model by adding a term to $\mathcal{L}_1'$ which contributes to the off-diagonal mass matrix elements with a phase different from the one already present. This can be incorporated for example by an additional term like $\delta K^0\eta\eta$ which amounts to a contribution to $\mathcal{M}_{21}$ with a phase $\arg 2\delta^2 \neq \arg(2\beta^2+\gamma^2)$. A simple model interaction with CP-violation both in mixing and in the amplitudes is then given by

$$\begin{aligned}
\mathcal{L}_2 &= \mathcal{L}_{free} + \mathcal{L}_2' \,,\\
\mathcal{L}_2' &= \beta K^0\pi^0\pi^0 + \gamma K^0\pi^+\pi^- + \delta K^0\eta\eta + h.c.\,. \qquad (4.33)
\end{aligned}$$

We will now determine the exponentially decaying states $|K_{S,L}\rangle$ via the mass matrix in order to calculate the CP-violating parameters induced by this model.

4.2.1 The mass matrix and CP-violation in mixing

The calculation of the mass matrix $\mathcal{M}$ is analogous to section 4.1. The new contribution to the self energy of the kaon coming from the additional term in (4.33) is given by

$$\begin{aligned}
\Sigma_{11}(p^2) = \Sigma_{22}(p^2) &= 2\overline{\delta}\delta\overline{J}(p^2;M_\eta,M_\eta)\,, \qquad (4.34)\\
\Sigma_{12}(p^2) &= 2\overline{\delta}^2\overline{J}(p^2;M_\eta,M_\eta)\,, \qquad (4.35)\\
\Sigma_{21}(p^2) &= 2\delta^2\overline{J}(p^2;M_\eta,M_\eta)\,. \qquad (4.36)
\end{aligned}$$

Thus we have for the mass matrix

$$\mathcal{M}_{11} = \mathcal{M}_{22} = M_K - (2\beta\overline{\beta} + \gamma\overline{\gamma})\frac{1}{2M_K}\overline{J}(M_K^2;M_\pi,M_\pi)$$

$$-\overline{\delta}\delta\frac{1}{M_K}\overline{J}(p^2;M_\eta,M_\eta)\,, \tag{4.37}$$

$$\mathcal{M}_{12} = -(2\overline{\beta}^2+\overline{\gamma}^2)\frac{1}{2M_K}\overline{J}(M_K^2;M_\pi,M_\pi)$$
$$-\overline{\delta}^2\frac{1}{M_K}\overline{J}(p^2;M_\eta,M_\eta) \tag{4.38}$$

$$\mathcal{M}_{21} = -(2\beta^2+\gamma^2)\frac{1}{2M_K}\overline{J}(M_K^2;M_\pi,M_\pi)$$
$$-\delta^2\frac{1}{M_K}\overline{J}(p^2;M_\eta,M_\eta)\,. \tag{4.39}$$

We can decompose the mass matrix in its hermitian parts according to (2.13):

$$M_{11}=M_{22} = M_K-(2\beta\overline{\beta}+\gamma\overline{\gamma})\frac{1}{2M_K}\mathrm{Re}\,\overline{J}(M_K^2;M_\pi,M_\pi)$$
$$-\delta\overline{\delta}\frac{1}{M_K}\overline{J}(M_K^2;M_\eta,M_\eta)\,, \tag{4.40}$$

$$M_{12}=\overline{M_{21}} = -(2\overline{\beta}^2+\overline{\gamma}^2)\frac{1}{2M_K}\mathrm{Re}\,\overline{J}(M_K^2;M_\pi,M_\pi)$$
$$-\overline{\delta}^2\frac{1}{M_K}\overline{J}(M_K^2;M_\eta,M_\eta)\,, \tag{4.41}$$

$$\Gamma_{11}=\Gamma_{22} = +(2\beta\overline{\beta}+\gamma\overline{\gamma})\frac{1}{M_K}\mathrm{Im}\,\overline{J}(M_K^2;M_\pi,M_\pi)\,, \tag{4.42}$$

$$\Gamma_{12}=\overline{\Gamma_{21}} = +(2\overline{\beta}^2+\overline{\gamma}^2)\frac{1}{M_K}\mathrm{Im}\,\overline{J}(M_K^2;M_\pi,M_\pi)\,, \tag{4.43}$$

since $\mathrm{Im}\,\overline{J}(M_K^2;M_\eta,M_\eta)=0$. The eigenvalues are again determined by equations (4.16) and (4.17). The K_L - eigenstate in the $K^0-\overline{K}^0$ -basis is then given by $(1,\tilde{\eta})$ with

$$\tilde{\eta}=\frac{Q}{\mathcal{M}_{12}}=\sqrt{\frac{2\delta^2\overline{J}(M_K^2;M_\eta,M_\eta)+(2\beta^2+\gamma^2)\overline{J}(M_K^2;M_\pi,M_\pi)}{2\overline{\delta}^2\overline{J}(M_K^2;M_\eta,M_\eta)+(2\overline{\beta}^2+\overline{\gamma}^2)\overline{J}(M_K^2;M_\pi,M_\pi)}}\,, \tag{4.44}$$

where the sign of the square root is determined through (2.20).

Now it is obvious that the lack of relative reality of $2\delta^2$ and $2\beta^2+\gamma^2$ is in fact responsible for CP-violation in mixing, since then $|\tilde{\eta}|\neq 1$ or $\arg\frac{\Gamma_{12}}{M_{12}}\neq 0$, and the states $|K_{L,S}\rangle$ are no longer orthogonal.

On the other hand, if $\arg 2\delta^2=\arg(2\beta^2+\gamma^2)$, we can easily see CP-invariance in mixing, since in this case $\tilde{\eta}$ becomes

$$\tilde{\eta}=-e^{i\arg 2\delta^2}\,, \tag{4.45}$$

which is again a pure phase that can be absorbed in the definition of the kaons by applying a strangeness transformation with a transformation angle $\alpha=\arg\delta\pm\frac{\pi}{2}$. We then have $\tilde{\eta}=1$ and

$$\rho=\frac{1-\tilde{\eta}}{1+\tilde{\eta}}=0\,, \tag{4.46}$$

which means in particular that the exponentially decaying states are the CP-eigenstates, $K_L\sim K_2$ and $K_S\sim K_1$, as in the previous model.

4.2.2 The $K \to 2\pi$-amplitudes and direct CP-violation

The additional term in (4.33) gives no contribution to the decay amplitudes of the kaon, so we can adopt the results calculated in section 4.1 for ω and ε'. However, the expressions for ρ and thus for ε, η_{00} and η_{+-} become very complicated, since $\tilde{\eta}$ is no longer a phase. Therefore we do not display the explicit expressions for η_{00} and η_{+-}, since it is not very illuminating. Nevertheless, the non-vanishing CP-violating quantities ε, ε' guarantee a non-zero η_{+-} and η_{00}, and indicate that (4.33) describes in general a model showing CP-violation both in mixing and in the amplitudes.

However, if $\varphi_\beta = \varphi_\gamma \bmod \pi$ we have no direct CP-violation, i.e. $\varepsilon' = 0$, and we can apply a strangeness transformation to the kaon fields in $\mathcal{L}'_2$ with a transformation angle $\alpha = \varphi_\beta - \frac{\pi}{2}$ in order to get (we assume $\varphi_\beta = \varphi_\gamma$)

$$\begin{aligned}
\mathcal{L}'_2 &= |\beta|(e^{i(\varphi_\beta-\alpha)}K^0_\alpha + e^{-i(\varphi_\beta-\alpha)}\overline{K}^0_\alpha)\pi^0\pi^0 \\
&\quad +|\gamma|(e^{i(\varphi_\beta-\alpha)}K^0_\alpha + e^{-i(\varphi_\beta-\alpha)}\overline{K}^0_\alpha)\pi^+\pi^- \\
&\quad +(e^{-i\alpha}\delta K^0_\alpha + e^{i\alpha}\overline{\delta}\,\overline{K}^0_\alpha)\eta\eta \qquad (4.47)\\
&= i(K^0_\alpha - \overline{K}^0_\alpha)(|\beta|\pi^0\pi^0 + |\gamma|\pi^+\pi^-) + (e^{-i\alpha}\delta K^0_\alpha + e^{i\alpha}\overline{\delta}\,\overline{K}^0_\alpha)\eta\eta\,, \qquad (4.48)
\end{aligned}$$

showing explicitely that only the CP-even combination $K_1 \sim (K^0 - \overline{K}^0)$ decays to two pions. Nevertheless, $\varepsilon \neq 0$ and therefore η_{00}, η_{+-} do not vanish.

We finally come to the conclusion that there is a necessary phase independent condition for each CP-violation in mixing and in the amplitudes. The latter requires a nonvanishing phase difference of the couplings that induce transitions of the kaon to the $2\pi^0$ and $\pi^+\pi^-$ states, i.e. two interfering amplitudes A_0 and A_2, while CP-violation in mixing requires at least two contributions to the self energy of the kaon each contributing with a different phase. The explicit form of the terms in the Lagrangian, which yield the additional contribution to the pion loops in the mass matrix, is of no importance. It can be a term like $\delta K^0\eta\eta$ as above or $\delta K^0\pi^0\eta$, as well as a combination of both.

4.3 A simple superweak model

In this section we will outline the features of a superweak model, i.e. a model that induces first order $\Delta S = 2$ transitions. We start with a Lagrangian inducing only transitions from K_1 to 2π-states. This can be realised by couplings of the kaon to $\pi^0\pi^0$ and $\pi^+\pi^-$ having the same phase modulo π, e.g.

$$\mathcal{L}_{int} = \beta K^0(\pi^0\pi^0 + \pi^+\pi^-) + h.c.\,. \qquad (4.49)$$

For this model we calculated $\varepsilon' = 0$ (see section 4.1) as required by a superweak model. CP-violation in mixing is now incorporated in the model by a term in the interaction inducing first order $\Delta S = 2$ transitions, e.g. $\delta K^0 K^0$. This leads to an interaction of the form

$$\mathcal{L}'_3 = \beta K^0(\pi^0\pi^0 + \pi^+\pi^-) + \delta K^0 K^0 + h.c.\,. \qquad (4.50)$$

In the calculation of the mass matrix we neglect terms quadratic in δ and we arrive at

$$\mathcal{M}_{11} = \mathcal{M}_{22} = M_K + 3\beta\overline{\beta}\frac{1}{2M_K}\overline{J}(M_K^2; M_\pi, M_\pi)\,, \qquad (4.51)$$

$$\mathcal{M}_{12} = 3\overline{\beta}^2 \frac{1}{2M_K} \overline{J}(M_K^2; M_\pi, M_\pi) + \frac{\overline{\delta}}{2M_K}, \tag{4.52}$$

$$\mathcal{M}_{21} = 3\beta^2 \frac{1}{2M_K} \overline{J}(M_K^2; M_\pi, M_\pi) + \frac{\delta}{2M_K}, \tag{4.53}$$

which leads to the situation $|\tilde{\eta}| \neq 1$, and hence CP-violation in mixing, if $\arg \delta \neq \arg \beta^2 \mod \pi$.

In this superweak model it is the same mechanism as in the previous sections that leads to CP-violation in mixing, that is two contributions to $\mathcal{M}$ contributing with different phases. The eigenvalues of the mass matrix $\mathcal{M}$ are again determined by equations (4.16) and (4.17). The K_L - eigenstate in the $K^0 - \overline{K}^0$ -basis is then given by $(1, \tilde{\eta})$ with

$$\tilde{\eta} = \frac{Q}{\mathcal{M}_{12}} = \sqrt{\frac{\delta + 3\beta^2 \overline{J}(M_K^2; M_\pi, M_\pi)}{\overline{\delta} + 3\overline{\beta}^2 \overline{J}(M_K^2; M_\pi, M_\pi)}}, \tag{4.54}$$

where the sign of the square root is again determined through (2.20). The expressions for η_{00} and η_{+-} are derived from (2.50), (2.51) and $\varepsilon' = 0$, and we obtain

$$\eta_{+-} = \eta_{00} = \varepsilon\,. \tag{4.55}$$

Chapter 5

The effective $\Delta S = 1$ nonleptonic weak interaction

5.1 Construction of the effective $\Delta S = 1$ nonleptonic weak Lagrangian $\mathcal{L}_{W1} = \mathcal{L}_{W1}^{8} + \mathcal{L}_{W1}^{27}$

The physical fields of the effective Lagrangian are the pseudoscalar mesons. They are contained in an unitary matrix, $U(x)$, which transforms under the chiral group, $SU(3)_R \times SU(3)_L$, as follows:

$$U \to V_R U V_L^{\dagger} , \tag{5.1}$$

where $V_{R,L}$ are elements of $SU(3)_{R,L}$. A convenient representation of U is the exponential one:

$$U(\phi) = \exp\left(i\frac{\phi}{F_\pi}\right) , \tag{5.2}$$

where

$$\phi = \sum_{a}^{8} \lambda_a \phi^a , \tag{5.3}$$

and F_π is a constant with the dimension of a mass. We are especially interested in the properties of U under the discrete transformations C, P and the combined one CP. Using the same phase convention as in (2.1) and considering the U-fields as classical c-number quantities one has:

$$U \xrightarrow{C} U^T , \tag{5.4}$$

$$U \xrightarrow{P} U^{\dagger} , \tag{5.5}$$

$$U \xrightarrow{CP} \overline{U} . \tag{5.6}$$

With the field matrix U we can build up two currents, L_μ and R_μ, which transform only under the left- or right-handed chiral group:

$$L_\mu = iU^{\dagger}\partial_\mu U \quad \to \quad V_L L_\mu V_L^{\dagger} , \tag{5.7}$$

$$R_\mu = iU\partial_\mu U^{\dagger} \quad \to \quad V_R R_\mu V_R^{\dagger} . \tag{5.8}$$

From the transformation properties of U and ∂_μ under the discrete transformations C, P and CP we can derive those of the currents:

$$L_\mu \overset{C}{\longleftrightarrow} -(R^\mu)^T , \tag{5.9}$$
$$L_\mu \overset{P}{\longleftrightarrow} R^\mu , \tag{5.10}$$
$$L_\mu \overset{CP}{\longleftrightarrow} -(L^\mu)^T , \tag{5.11}$$
$$R_\mu \overset{CP}{\longleftrightarrow} -(R^\mu)^T . \tag{5.12}$$

The lowest-order effective chiral lagrangian describing the non-leptonic strong interactions is dictated by the requirements of Lorentz and chiral invariance as well as under parity inversion and charge conjugation:

$$\mathcal{L}_{st} = \frac{F_\pi^2}{4} \left\{ \langle \partial_\mu U^\dagger \partial^\mu U \rangle + \langle U^\dagger M + M^\dagger U \rangle \right\} , \tag{5.13}$$

where the brackets $\langle\,\rangle$ denote the trace in the flavour space of 3×3 matrices and M is the quark mass matrix transforming under the chiral group exactly as U does:

$$M = 2B_0 \begin{pmatrix} \hat{m} & & \\ & \hat{m} & \\ & & m_s \end{pmatrix} , \tag{5.14}$$

where B_0 is a constant with the dimension of a mass and $\hat{m} = \frac{1}{2}(m_u + m_d)$ corresponds to the isospin limit.

Now the lowest-order effective lagrangian describing the non-leptonic weak $\Delta S = 1$ interactions is constructed [5] again according to the symmetries of the corresponding interaction. From the standard model one knows that the weak interaction arises from a symmetric product of left-chiral currents which are the charged members of an octet:

$$\mathcal{L}_{\Delta S=1} = g \left\{ (J_{1\mu} + iJ_{2\mu})(J_4^\mu + iJ_5^\mu) + (J_{4\mu} + iJ_{5\mu})(J_1^\mu + iJ_2^\mu) \right\} + h.c.. \tag{5.15}$$

From this it follows that the nonleptonic interaction transforms as $(27_L, 1_R) \oplus (8_L, 1_R)$ under the chiral group.

In order to construct the operators of order p^2 with the required symmetry properties it is useful to work with non-hermitian tensor matrices Q_a^b instead of the hermitian Gell-Mann matrices. They are defined as follows:

$$(Q_a^b)_{ij} = \delta_{ai}\delta_{bj} - \frac{1}{3}\delta_{ab}\delta_{ij} , \tag{5.16}$$

and project out the corresponding octet components of a hermitian traceless 3×3 matrix $\mathcal{P}$ via the trace:

$$\langle Q_a^b \mathcal{P} \rangle = \mathcal{P}_{ba} . \tag{5.17}$$

Thus we can construct two hermitian octet operators with the required transformation properties yielding the octet part of the Lagrangian [13],

$$\mathcal{L}_{W1}^{8+} \sim \langle (Q_3^2 + Q_2^3) L_\mu L^\mu \rangle , \tag{5.18}$$
$$\mathcal{L}_{W1}^{8-} \sim i\langle (Q_3^2 - Q_2^3) L_\mu L^\mu \rangle , \tag{5.19}$$

where the superscript $\pm$ shows here and in the following the transformation property under the CP-transformation as defined in (2.1). The octet character of the two operators is manifest when they are rewritten in terms of Gell-Mann matrices,

$$Q_3^2 + Q_2^3 = \lambda_6 \,, \tag{5.20}$$

$$i(Q_3^2 - Q_2^3) = \lambda_7 \,. \tag{5.21}$$

The operators transforming as $(27_L, 1_R)$ are constructed as the irreducible products of the octet components of L_μ. In the 27-plet there are two operators with quantum numbers $\Delta S = 1$, $\Delta Q = 0$, one belonging to the $(I = \frac{1}{2})$-doublet, the other to the $(I = \frac{3}{2})$-quadruplet.

The hermitian combinations of these operators yield

$$\begin{aligned} \mathcal{L}_{W1}^{27+}(\Delta I = \frac{1}{2}) &\sim \langle Q_3^1 L_\mu\rangle\langle Q_1^2 L^\mu\rangle + \langle Q_1^3 L_\mu\rangle\langle Q_2^1 L^\mu\rangle \\ &\quad + \langle (Q_3^2 + Q_2^3) L_\mu\rangle \left\{ 4\langle Q_1^1 L^\mu\rangle + 5\langle Q_2^2 L^\mu\rangle \right\} \,, \end{aligned} \tag{5.22}$$

$$\begin{aligned} \mathcal{L}_{W1}^{27-}(\Delta I = \frac{1}{2}) &\sim i\left\{ \langle Q_3^1 L_\mu\rangle\langle Q_1^2 L^\mu\rangle - \langle Q_1^3 L_\mu\rangle\langle Q_2^1 L^\mu\rangle \right\} \\ &\quad + i\langle (Q_3^2 - Q_2^3) L_\mu\rangle \left\{ 4\langle Q_1^1 L^\mu\rangle + 5\langle Q_2^2 L^\mu\rangle \right\} \,, \end{aligned} \tag{5.23}$$

$$\begin{aligned} \mathcal{L}_{W1}^{27+}(\Delta I = \frac{3}{2}) &\sim \langle Q_3^1 L_\mu\rangle\langle Q_1^2 L^\mu\rangle + \langle Q_1^3 L_\mu\rangle\langle Q_2^1 L^\mu\rangle \\ &\quad + \langle (Q_3^2 + Q_2^3) L_\mu\rangle \left\{ \langle Q_1^1 L^\mu\rangle - \langle Q_2^2 L^\mu\rangle \right\} \,, \end{aligned} \tag{5.24}$$

$$\begin{aligned} \mathcal{L}_{W1}^{27-}(\Delta I = \frac{3}{2}) &\sim i\left\{ \langle Q_3^1 L_\mu\rangle\langle Q_1^2 L^\mu\rangle - \langle Q_1^3 L_\mu\rangle\langle Q_2^1 L^\mu\rangle \right\} \\ &\quad + i\langle (Q_3^2 - Q_2^3) L_\mu\rangle \left\{ \langle Q_1^1 L^\mu\rangle - \langle Q_2^2 L^\mu\rangle \right\} \,. \end{aligned} \tag{5.25}$$

According to the authors of [14] the 27-plet operators are related by

$$\mathcal{L}_{W1}^{27\pm} \sim \frac{1}{3}\mathcal{L}_{W1}^{27\pm}(\Delta I = \frac{1}{2}) + \frac{5}{3}\mathcal{L}_{W1}^{27\pm}(\Delta I = \frac{3}{2}) \tag{5.26}$$

in the $SU(3)$-limit, yielding

$$\begin{aligned} \mathcal{L}_{W1}^{27+} &\sim 3\langle (Q_3^2 + Q_2^3) L_\mu\rangle\langle Q_1^1 L^\mu\rangle \\ &\quad + 2\left\{ \langle Q_3^1 L_\mu\rangle\langle Q_1^2 L^\mu\rangle + \langle Q_1^3 L_\mu\rangle\langle Q_2^1 L^\mu\rangle \right\} \,, \end{aligned} \tag{5.27}$$

$$\begin{aligned} \mathcal{L}_{W1}^{27-} &\sim 3i\langle (Q_3^2 - Q_2^3) L_\mu\rangle\langle Q_1^1 L^\mu\rangle \\ &\quad + 2i\left\{ \langle Q_3^1 L_\mu\rangle\langle Q_1^2 L^\mu\rangle - \langle Q_1^3 L_\mu\rangle\langle Q_2^1 L^\mu\rangle \right\} \,. \end{aligned} \tag{5.28}$$

Thus the 27-plet lagrangians, $\mathcal{L}_{W1}^{27+}$ and $\mathcal{L}_{W1}^{27-}$, induce both $|\Delta I| = \frac{1}{2}$ and $|\Delta I| = \frac{3}{2}$ transitions via its components (5.22) - (5.25).

It is convenient to write down the parts of the Lagrangian in a compact notation. For the octet part we can write

$$\mathcal{L}_{W1}^{8} = F_\pi^4 C_8 f_b^a \langle Q_a^b L_\mu L^\mu\rangle \,, \tag{5.29}$$

where we deduce $\mathcal{L}_{W1}^{8+}$ by setting $C_8 = c_2^+$, $f_2^3 = f_3^2 = 1$ and $f_i^j = 0$ otherwise. $\mathcal{L}_{W1}^{8-}$ follows from putting $C_8 = c_2^-$, $f_2^3 = i$, $f_3^2 = -i$ and $f_i^j = 0$ otherwise.

The 27-plet part is given by the tensor

$$\mathcal{L}_{W1}^{27} = F_\pi^4 C_{27} t_{cd}^{ab} \langle Q_a^c L_\mu \rangle \langle Q_b^d L^\mu \rangle \,, \tag{5.30}$$

where we obtain the different parts (5.22) - (5.28) by setting t_{cd}^{ab} according to Table 5.1.

Table 5.1: Values of the tensor coefficients for the different parts of the 27-plet. All other t_{cd}^{ab} are equal to zero.

	C_{27}	t_{12}^{31}	t_{31}^{12}	t_{21}^{31}	t_{31}^{21}	t_{22}^{32}	t_{32}^{22}
$\mathcal{L}_{W1}^{27+}$	c_3^+	2	2	3	3	0	0
$\mathcal{L}_{W1}^{27-}$	c_3^-	2i	-2i	3i	-3i	0	0
$\mathcal{L}_{W1}^{27+}(1/2)$	$c_3^{\prime +}$	1	1	4	4	5	5
$\mathcal{L}_{W1}^{27-}(1/2)$	$c_3^{\prime -}$	i	-i	4i	-4i	5i	-5i
$\mathcal{L}_{W1}^{27+}(3/2)$	$c_3^{\prime\prime +}$	1	1	1	1	-1	-1
$\mathcal{L}_{W1}^{27-}(3/2)$	$c_3^{\prime\prime -}$	i	-i	i	-i	-i	i

$c_2^\pm$ and $c_3^\pm$, $c_3^{\prime\pm}$, $c_3^{\prime\prime\pm}$ are free real parameters of order G_F, which cannot be determined by symmetry arguments alone. They must be calculated from a more fundamental theory or determined from experiment. However, as already stated, in $SU(3)$-limit, $c_3^{\prime\pm}$ and $c_3^{\prime\prime\pm}$ are related by

$$c_3^{\prime\pm} = \frac{1}{3} c_3^\pm \quad \text{and} \quad c_3^{\prime\prime\pm} = \frac{5}{3} c_3^\pm \,. \tag{5.31}$$

Now we are able to write down the complete effective $\Delta S = 1$ nonleptonic weak interaction Lagrangian in its general form [5]:

$$\mathcal{L}_{W1} = \mathcal{L}_{W1}^{8+} + \mathcal{L}_{W1}^{8-} + \mathcal{L}_{W1}^{27+} + \mathcal{L}_{W1}^{27-} \,. \tag{5.32}$$

5.2 CP-invariance of $\mathcal{L}_{W1}^{8}$ and $\mathcal{L}_{W1}^{27}$

In this section we will show that $\mathcal{L}_{W1}^{8}$ and $\mathcal{L}_{W1}^{27}$ are CP-invariant. We will give the strangeness transformation angle α by which we redefine the fields in order to make the CP-invariance explicit. The same angle can be used to redefine the phase of the CP-transformation in (2.1), as we mentioned in section 2.4.

In order to look for possible CP-violation in $\mathcal{L}_{W1}^{8}$ and $\mathcal{L}_{W1}^{27}$ we have to determine the exponentially decaying states via the mass matrix. Thus we have to calculate the self energy $\Sigma(M_K^2)$ of the kaon up to one loop. The contributing Feynman diagrams are shown in figure 5.1. In appendix B.2 we present the results of some contributions (see figure B.1 for details).

CP-violation in mixing shows up in a deviation from $|\tilde{\eta}| = 1$, where $\tilde{\eta}$ is given by (2.24). Expanding U in the Lagrangians $\mathcal{L}_{W1}^{8}$ and $\mathcal{L}_{W1}^{27}$ to the appropriate order in the fields, we see that the K^0 couples only with $(c_2^- - ic_2^+)$ and $(c_3^- - ic_3^+)$, respectively, with the result that all diagrams in figure 5.1 contribute with the same phase to the off- diagonal element $\Sigma_{12}(M_K^2)$ of the total self energy matrix:

$$\mathcal{L}_{W1}^{8} \quad \rightarrow \quad \begin{cases} \Sigma_{12}(M_K^2) = (c_2^- + ic_2^+)^2 \cdot P_8(M_K^2) \\ \Sigma_{21}(M_K^2) = (c_2^- - ic_2^+)^2 \cdot P_8(M_K^2) \end{cases} , \tag{5.33}$$

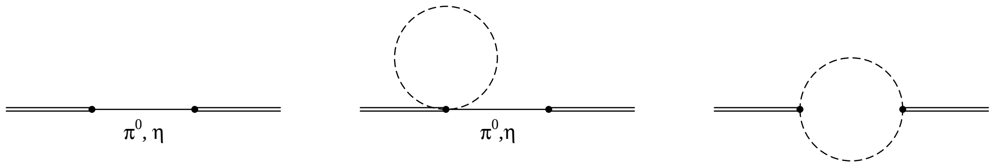

Figure 5.1: Contributions to the self energy of the kaon. A simple line represents a pion or an eta, whereas a double line denotes a kaon. The dashed loops consist of virtual K^+K^-, $K^0\overline{K}^0$, $\pi^+\pi^-$, $\pi^0\pi^0$, $\eta\eta$ or $\pi^0\eta$-pairs.

$$\mathcal{L}_{W1}^{27} \quad \to \quad \left\{ \begin{array}{l} \Sigma_{12}(M_K^2) = (c_3^- + ic_3^+)^2 \cdot P_{27}(M_K^2) \\ \Sigma_{21}(M_K^2) = (c_3^- - ic_3^+)^2 \cdot P_{27}(M_K^2) \end{array} \right. , \tag{5.34}$$

where $P_8(M_K^2)$ and $P_{27}(M_K^2)$ are some complex functions. Therefore we get from (2.24)

$$\mathcal{L}_{W1}^{8} \quad \to \quad \tilde{\eta} = e^{i2\varphi_2} , \tag{5.35}$$

$$\mathcal{L}_{W1}^{27} \quad \to \quad \tilde{\eta} = e^{i2\varphi_3} , \tag{5.36}$$

where $\varphi_2 = \arg(c_2^+ + ic_2^-)$ and $\varphi_3 = \arg(c_3^+ + ic_3^-)$, respectively, and the sign of $\tilde{\eta}$ is determined through (2.20). Since $\tilde{\eta}$ is only a phase for both $\mathcal{L}_{W1}^{8}$ and $\mathcal{L}_{W1}^{27}$, we can transform it away with a suitable redefinition of the kaon fields by a strangeness transformation in order to get $\tilde{\eta} = 1$, which shows that the exponentially decaying states are $|K_S\rangle = |K_1\rangle$ and $|K_L\rangle = |K_2\rangle$, i.e. we have CP-invariance in mixing.

Furthermore we obtain from (B.10), (B.11) and table B.1

$$\mathcal{L}_{W1}^{8} \quad \to \quad \arg A_0 = \varphi_2 \pm \pi , \tag{5.37}$$

$$\mathcal{L}_{W1}^{27} \quad \to \quad \arg A_0 = \arg A_2 = \varphi_3 \pm \pi , \tag{5.38}$$

which is an evident sign of CP-invariance in the amplitudes.

The CP-invariance to all orders can be seen explicitely in the Lagrangians $\mathcal{L}_{W1}^{8}$ and $\mathcal{L}_{W1}^{27}$ if we use a strangeness transformation to redefine the relative phase of the kaon fields.

In tensor notation (5.29) and (5.30) we have $L_\mu^\alpha = e^{-i\alpha S} L_\mu e^{+i\alpha S}$, where $S = -Q_3^3$ in the 3-dimensional representation of $SU(3)$.

This yields for the octet part, writing $c_i^+ + ic_i^- = |c_i^+ + ic_i^-| \cdot e^{i\varphi_i}$, $(i = 2, 3)$:

$$\begin{aligned} \mathcal{L}_{W1}^{8} &= |c_2^+ + ic_2^-| \langle \left(e^{+i\varphi_2} Q_3^2 + e^{-i\varphi_2} Q_2^3 \right) L_\mu L^\mu \rangle \\ &= |c_2^+ + ic_2^-| \langle e^{-i\alpha S} \left(e^{+i\varphi_2} Q_3^2 + e^{-i\varphi_2)} Q_2^3 \right) e^{+i\alpha S} L_\mu^\alpha L_\alpha^\mu \rangle , \end{aligned} \tag{5.39}$$

where we used the invariance of cyclic permutations of matrices under the trace. Since

$$[S, Q_3^2] = -Q_3^2 \quad \text{and} \quad [S, Q_2^3] = +Q_2^3 , \tag{5.40}$$

we have

$$e^{-i\alpha S} Q_3^2 e^{+i\alpha S} \quad = \quad e^{+i\alpha} Q_3^2 , \tag{5.41}$$

$$e^{-i\alpha S} Q_2^3 e^{+i\alpha S} \quad = \quad e^{-i\alpha} Q_2^3 , \tag{5.42}$$

and thus

$$\mathcal{L}_{W1}^{8} = |c_2^+ + ic_2^-| \langle \left(e^{+i(\varphi_2+\alpha)} Q_3^2 + e^{-i(\varphi_2+\alpha)} Q_2^3 \right) L_\mu^\alpha L_\alpha^\mu \rangle \, . \tag{5.43}$$

Choosing $\alpha = -\varphi_2$ and omitting the sub- and superscript α the complete 'strangeness-transformed' octet Lagrangian reads in tensor notation:

$$\mathcal{L}_{W1}^{8} = |c_2^+ + ic_2^-| \langle \left(Q_3^2 + Q_2^3 \right) L_\mu L^\mu \rangle \, , \tag{5.44}$$

showing the CP-invariance to all orders explicitely. Note, that the CP-transformation is still defined according to (2.1) and (5.11), respectively.

A similar argumentation is valid for the 27-plet part of the lagrangian, $\mathcal{L}_{W1}^{27}$:

$$\begin{aligned}
\mathcal{L}_{W1}^{27} &= |c_3^+ + ic_3^-| \Big(3\langle e^{+i\varphi_3} Q_3^2 L_\mu \rangle \langle Q_1^1 L^\mu \rangle \\
&\quad + 2\langle e^{+i\varphi_3} Q_3^1 L_\mu \rangle \langle Q_1^2 L^\mu \rangle \Big) + h.c. \qquad (5.45) \\
&= |c_3^+ + ic_3^-| \Big(3\langle e^{+i\varphi_3} e^{-i\alpha S} Q_3^2 e^{+i\alpha S} L_\mu^\alpha \rangle \langle e^{-i\alpha S} Q_1^1 e^{+i\alpha S} L_\alpha^\mu \rangle \\
&\quad + 2\langle e^{+i\varphi_3} e^{-i\alpha S} Q_3^1 e^{+i\alpha S} L_\mu^\alpha \rangle \langle e^{-i\alpha S} Q_1^2 e^{+i\alpha S} L_\alpha^\mu \rangle \Big) + h.c. \, . \qquad (5.46)
\end{aligned}$$

In addition to (5.40) we have

$$[S, Q_1^3] = +Q_1^3 \, , \tag{5.47}$$

$$[S, Q_3^1] = -Q_3^1 \, , \tag{5.48}$$

$$[S, Q_1^2] = [S, Q_2^1] = [S, Q_1^1] = 0 \, , \tag{5.49}$$

and thus

$$e^{-i\alpha S} Q_3^1 e^{+i\alpha S} = e^{+i\alpha} Q_3^1 \, , \tag{5.50}$$

$$e^{-i\alpha S} Q_1^3 e^{+i\alpha S} = e^{-i\alpha} Q_1^3 \, , \tag{5.51}$$

while Q_1^2, Q_2^1 and Q_1^1 remain invariant, and we obtain

$$\begin{aligned}
\mathcal{L}_{W1}^{27} &= |c_3^+ + ic_3^-| \Big(3\langle e^{+i(\varphi_3+\alpha)} Q_3^2 L_\mu^\alpha \rangle \langle Q_1^1 L_\alpha^\mu \rangle \\
&\quad + 2\,\langle e^{+i(\varphi_3+\alpha)} Q_3^1 L_\mu^\alpha \rangle \langle Q_1^2 L_\alpha^\mu \rangle \Big) + h.c. \, . \qquad (5.52)
\end{aligned}$$

Choosing $\alpha = -\varphi_3$ and omitting the sub- and superscript α this becomes

$$\begin{aligned}
\mathcal{L}_{W1}^{27} &= |c_3^+ + ic_3^-| \Big(3\langle (Q_3^2 + Q_2^3) L_\mu \rangle \langle Q_1^1 L^\mu \rangle \\
&\quad + 2\,\Big\{ \langle Q_3^1 L_\mu \rangle \langle Q_1^2 L^\mu \rangle + \langle Q_1^3 L_\mu \rangle \langle Q_2^1 L^\mu \rangle \Big\} \Big) \, , \qquad (5.53)
\end{aligned}$$

showing the CP-invariance of the complete 27-plet Lagrangian explicitely to all orders.

As we already mentioned in section 2.4 this redefinition of the relative phases of the kaon fields is in fact equivalent to a redefinition of the CP-transformation phase in (2.1) and (5.11), respectively.

5.3 CP-invariance of $\mathcal{L}_{W1}^{-} = \mathcal{L}_{W1}^{8-} + \mathcal{L}_{W1}^{27-}$

In this section we will show that $\mathcal{L}_{W1}^{-} = \mathcal{L}_{W1}^{8-} + \mathcal{L}_{W1}^{27-}$ is CP-invariant to all orders. By looking at $\mathcal{L}_{W1}^{-}$ it is obvious that $\mathcal{L}_{W1}^{8-}$ and $\mathcal{L}_{W1}^{27-}$ contribute with the same phase to the off-diagonal self energy matrix elements, since the phase of the couplings is $\pm\frac{\pi}{2}$ for both $\mathcal{L}_{W1}^{8-}$ and $\mathcal{L}_{W1}^{27-}$, which leads to $\arg\frac{\Gamma_{12}}{M_{12}} = 0 \mod \pi$ reflecting CP-invariance in mixing.

Furthermore, from (B.10), (B.11) and table B.1, it is obvious that $\arg A_0 = \arg A_2 \mod \pi$, which is equivalent to CP-invariance in the decay amplitudes.

From (5.29), (5.30) and table 5.1 we have

$$\begin{aligned}\mathcal{L}_{W1}^{-} &= 3c_3^{-}e^{i\frac{\pi}{2}}\langle Q_3^2 L_\mu\rangle\langle Q_1^1 L^\mu\rangle + 2c_3^{-}e^{i\frac{\pi}{2}}\langle Q_3^1 L_\mu\rangle\langle Q_1^2 L^\mu\rangle \\ &\quad + c_2^{-}e^{i\frac{\pi}{2}}\langle Q_3^2 L_\mu L^\mu\rangle + h.c.\,. \end{aligned} \tag{5.54}$$

Redefining the fields with a strangeness transformation with $\alpha = -\frac{\pi}{2}$ and using again the commutation relations of the tensors Q_a^b with $S = -Q_3^3$ as in the previous section, we obtain:

$$\begin{aligned}\mathcal{L}_{W1}^{-} &= 3|c_3^{-}|\langle Q_3^2 L_\mu\rangle\langle Q_1^1 L^\mu\rangle + 2|c_3^{-}|\langle Q_3^1 L_\mu\rangle\langle Q_1^2 L^\mu\rangle \\ &\quad + |c_2^{-}|\langle Q_3^2 L_\mu L^\mu\rangle + h.c.\,, \end{aligned} \tag{5.55}$$

where we have omitted again the sub- and superscripts of L_μ and L^μ. From (5.11) it is obvious that $\mathcal{L}_{W1}^{-}$ is in fact CP-even:

$$(CP)\mathcal{L}_{W1}^{-}(CP)^\dagger = +\mathcal{L}_{W1}^{-}\,, \tag{5.56}$$

where the CP-transformation on the new fields and currents is defined according to (2.1) and (5.11), respectively.

5.4 CP-violation in $\mathcal{L}_{W1} = \mathcal{L}_{W1}^{8} + \mathcal{L}_{W1}^{27}$

CP-violation in $\mathcal{L}_{W1} = \mathcal{L}_{W1}^{8} + \mathcal{L}_{W1}^{27}$ is reflected by the fact that none of the two necessary conditions for CP-invariance, (2.33) and (2.40), is fulfilled.

CP-violation in mixing shows up in the fact that we have contributions to the self energy contributing with different phases (see (B.28) and (B.31)) to M_{12} and Γ_{12}, if $\varphi_2 \neq \varphi_3 \mod \pi$. This leads to the statement

$$\arg\frac{\Gamma_{12}}{M_{12}} \neq 0 \mod \pi\,, \tag{5.57}$$

which is sufficient for CP-violation in mixing.

On the other hand CP-violation in the amplitudes can be seen by looking at the decay amplitudes A_0 and A_2 in (B.10) and (B.11), respectively. From table B.1 we find that, whenever $\varphi_2 \neq \varphi_3 \mod \pi$,

$$\arg A_0 \neq \arg A_2 \mod \pi\,, \tag{5.58}$$

which is a sufficient condition for direct CP-violation.

These considerations are equivalent to the statement, that there exists no phase α for which the Lagrangian $\mathcal{L}_{W1}(\phi) = \mathcal{L}^{\alpha}_{W1}(\phi_\alpha)$ is invariant under $(CP)_\alpha$ (see section 2.4), whenever $\varphi_3 \neq \varphi_2$ mod π. This can be seen if we rewrite the octet part as

$$\begin{aligned}\langle Q^2_3 L_\mu L^\mu\rangle &= \langle Q^1_3 L_\mu\rangle\langle Q^2_1 L^\mu\rangle + \langle Q^2_3 L_\mu\rangle\langle Q^2_2 L^\mu\rangle + \langle Q^3_3 L_\mu\rangle\langle Q^2_3 L^\mu\rangle \qquad (5.59)\\ &= \langle Q^1_3 L_\mu\rangle\langle Q^2_1 L^\mu\rangle - \langle Q^2_3 L_\mu\rangle\langle Q^1_1 L^\mu\rangle\,, \qquad (5.60)\end{aligned}$$

where it is explicit, that the operator is the irreducible octet part of the product of two octet currents. In the last step we have used the $SU(3)$-identity

$$Q^1_1 + Q^2_2 + Q^3_3 = 0\,. \qquad (5.61)$$

Now the complete Lagrangian $\mathcal{L}_{W1}$ reads

$$\mathcal{L}_{W1} = (3c_3 - c_2)\langle Q^2_3 L_\mu\rangle\langle Q^1_1 L^\mu\rangle + (2c_3 + c_2)i\langle Q^1_3 L_\mu\rangle\langle Q^2_1 L^\mu\rangle + h.c.\,, \qquad (5.62)$$

where $c_j = c^+_j + ic^-_j$, $j = 2, 3$. In order to make the CP-invariance explicit, it is obvious that we have to change the relative phase of the kaons with the help of a strangeness transformation with $\alpha = -\arg(3c_3 - c_2)$ in the first term, whereas the second term requires $\alpha = -\arg(2c_3 + c_2)$. However, this can not be fulfilled as long as $\varphi_3 \neq \varphi_2$ mod π. Thus we conclude that $\mathcal{L}_{W1}$ is in fact CP-violating, both in mixing and in the amplitudes.

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Appendix A

The Wigner-Weisskopf formalism

In 1930 Wigner and Weisskopf evolved a method that allows one to calculate the time evolution of a system of unstable states [9]. In this chapter we will outline the general formalism and point out the approximations made in the approach of Wigner and Weisskopf. We will strongly hold to the exposition of Nachtmann [6].

A.1 General formalism

We begin by considering a system which is described by a Hamiltonian

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}' , \tag{A.1}$$

where $\mathcal{H}_0$ is the free Hamiltonian of the system and $\mathcal{H}'$ is a small perturbation. The eigenstates of $\mathcal{H}_0$ are n degenerate energy states $|\alpha\rangle$ and a set of continuous states $|\beta\rangle$:

$$\mathcal{H}_0|\alpha\rangle = E_0|\alpha\rangle, \qquad (\alpha = 1, \ldots, n), \tag{A.2}$$

$$\mathcal{H}_0|\beta\rangle = E_\beta|\beta\rangle . \tag{A.3}$$

The small perturbation $\mathcal{H}'$ is responsible for the decay of the discrete states $|\alpha\rangle$ into the continuous states. Any given state can then be written as a superposition of the eigenstates $|\alpha\rangle$ and $|\beta\rangle$. When we consider a state at time $t = 0$ consisting only of the discrete states $|\alpha\rangle$, we are interested in the time development of such a state. For the time evolution of the states $|\alpha\rangle$ we would expect an expontial time dependence law. The Wigner-Weisskopf method shows that this in fact the case if we use some approximations. However, for very small and very large time scales this is not true and the states decay in a non-exponential manner.

The time evolution of a state

$$|t\rangle = \sum_{\alpha=1}^{n} a_\alpha(t)|\alpha\rangle + \sum_\beta b_\beta(t)|\beta\rangle \tag{A.4}$$

is given by a Schrödinger equation and is best described in the interaction picture:

$$i\frac{\partial}{\partial t}|t\rangle = \mathcal{H}'(t)|t\rangle , \tag{A.5}$$

where

$$\mathcal{H}'(t) = e^{i\mathcal{H}_0 t}\mathcal{H}'(0)e^{-i\mathcal{H}_0 t} \quad \text{and} \quad \mathcal{H}'(0) = \int d^3x \mathcal{H}'(0,\vec{x})\,. \tag{A.6}$$

Writing down (A.5) in components it reads

$$i\frac{\partial a_\alpha(t)}{\partial t} = \sum_{\alpha'}\langle\alpha|\mathcal{H}'|\alpha'\rangle a_{\alpha'}(t) + \sum_\beta e^{i(E_0-E_\beta)t}\langle\alpha|\mathcal{H}'|\beta\rangle b_\beta(t) \tag{A.7}$$

$$i\frac{\partial b_\beta(t)}{\partial t} = \sum_{\alpha'} e^{i(E_\beta-E_0)t}\langle\beta|\mathcal{H}'|\alpha'\rangle a_{\alpha'}(t) + \sum_{\beta'} e^{i(E_\beta-E_{\beta'})t}\langle\beta|\mathcal{H}'|\beta'\rangle b_{\beta'}(t)\,. \tag{A.8}$$

In order to solve equations (A.7) and (A.8) we have to use a first approximation, that is we neglect the second sum in (A.8). This means in particular that the continuous states $|\beta\rangle$ are not governed by the interaction Hamiltonian $\mathcal{H}'$, i.e. $\langle\beta|\mathcal{H}'|\beta'\rangle = 0$, and are thus stable states.

Using the Randbedingungen $a_\alpha(0) = a_\alpha^{(0)}$ and $b_\beta(0) = 0$ we can derive the two solutions

$$b_\beta(t) = -i\sum_{\alpha'}\int_0^t dt' e^{i(E_\beta-E_0)t'}\langle\beta|\mathcal{H}'|\alpha'\rangle a_{\alpha'}(t') \tag{A.9}$$

$$\begin{aligned} a_\alpha(t) &= a_\alpha^{(0)} - i\sum_{\alpha'}\int_0^t dt'\langle\alpha|\mathcal{H}'|\alpha'\rangle a_{\alpha'}(t') \\ &\quad -\sum_{\beta,\alpha'}\int_0^t dt'\int_0^{t'} dt'' e^{i(E_0-E_\beta)(t'-t'')}\langle\alpha|\mathcal{H}'|\beta\rangle\langle\beta|\mathcal{H}'|\alpha'\rangle a_{\alpha'}(t'')\,, \end{aligned} \tag{A.10}$$

in which the amplitudes a_α and b_β are no longer coupled. That allows one to solve (A.10) with the help of a Laplace transformation L_σ

$$\tilde{a}_\alpha(\sigma) = \int_0^\infty dt e^{-\sigma t} a_\alpha(t)\,. \tag{A.11}$$

Using

$$\mathrm{L}_\sigma\,[\text{const.}] = \frac{\text{const.}}{\sigma} \tag{A.12}$$

$$\mathrm{L}_\sigma\left[\int_0^t dt' f(t')\right] = \frac{1}{\sigma}\mathrm{L}_\sigma[f(t)] \tag{A.13}$$

$$\mathrm{L}_\sigma\left[e^{-at}f(t)\right] = \mathrm{L}_{\sigma+a}[f(t)] \tag{A.14}$$

one obtains

$$\tilde{a}_\alpha(\sigma) = \frac{a_\alpha^{(0)}}{\sigma} - \frac{i}{\sigma}\sum_{\alpha'}\mathcal{W}_{\alpha\alpha'}(\sigma)\tilde{a}_{\alpha'}(\sigma)\,, \tag{A.15}$$

where

$$\mathcal{W}_{\alpha\alpha'}(\sigma) = \langle\alpha|\mathcal{H}'|\alpha'\rangle + \sum_\beta \frac{\langle\alpha|\mathcal{H}'|\beta\rangle\langle\beta|\mathcal{H}'|\alpha'\rangle}{E_0 - E_\beta + i\sigma}\,. \tag{A.16}$$

Reading $\tilde{a}_\alpha$ as a vector and $\mathcal{W}_{\alpha\alpha'}$ as a matrix, we can solve (A.15):

$$\tilde{a}(\sigma) = (\sigma + i\mathcal{W}(\sigma))^{-1}a^{(0)}\,. \tag{A.17}$$

One can show that $(\sigma + i\mathcal{W}(\sigma))^{-1}$ is regular for Re $\sigma \neq 0$, but contains poles on the imaginary σ-axis. In order to obtain $a(t)$ from (A.17) we apply an inverse Laplace transformation to $\tilde{a}(\sigma)$ yielding

$$a(t) = \frac{1}{2\pi i}\int_{\varepsilon-i\infty}^{\varepsilon+i\infty} d\sigma e^{\sigma t}\frac{1}{\sigma + i\mathcal{W}}a^{(0)} \tag{A.18}$$

If we put $\mathcal{H}' = 0$ we have $\mathcal{W}(\sigma) = 0$ and thus a pole at $\sigma = 0$. The only contribution to the integral is then apparently coming from the pole at $\sigma = 0$ and leads to

$$a(t) = a^{(0)} \quad \text{for}\, t \geq 0\,. \tag{A.19}$$

The second approximation in the Wigner-Weisskopf method is now to assume that, for $\mathcal{H}' \neq 0$, the main contribution to the integral in (A.18) is still coming from the vicinity of the pole at $\sigma = 0$. This is in fact reasonable, since $\mathcal{H}'$ is assumed to be a small perturbation compared to $\mathcal{H}_0$. Therefore we can take $\mathcal{W}(\sigma)$ to be constant in the neighbourhood of $\sigma = 0$, $\mathrm{Re}\,\sigma > 0$

$$\mathcal{W}(\sigma) \to \mathcal{W} \equiv \lim_{\sigma\to+0}\mathcal{W}(\sigma)\,. \tag{A.20}$$

In this limes we obtain from (A.16)

$$\begin{aligned}\mathcal{W}_{\alpha\alpha'}(\sigma) \;&=\; \langle\alpha|\mathcal{H}'|\alpha'\rangle + \mathcal{P}\sum_{\beta}\frac{\langle\alpha|\mathcal{H}'|\beta\rangle\langle\beta|\mathcal{H}'|\alpha'\rangle}{(E_0 - E_\beta)} \\ &\quad -i\pi\sum_{\beta}\delta(E_0 - E_\beta)\langle\alpha|\mathcal{H}'|\beta\rangle\langle\beta|\mathcal{H}'|\alpha'\rangle\,,\end{aligned} \tag{A.21}$$

where $\mathcal{P}$ means the principal value, and we get using residuum calculus for $t > 0$

$$a(t) = \frac{1}{2\pi i}\int_{\varepsilon-i\infty}^{\varepsilon+i\infty} d\sigma e^{\sigma t}\frac{1}{\sigma + i\mathcal{W}}a^{(0)} = e^{-i\mathcal{W}t}a^{(0)}\,. \tag{A.22}$$

Changing now from the interaction picture to the Schrödinger picture, $|t\rangle_I = e^{i\mathcal{H}_0}|t\rangle_S$, we can write down the developement in time of the amplitudes $a(t)$ in (A.4):

$$a(t) = e^{-i\mathcal{M}t}a^{(0)} = e^{-i(E_0+\mathcal{W})t}a^{(0)}\,, \tag{A.23}$$

where

$$\mathcal{M} = M - \frac{i}{2}\Gamma \tag{A.24}$$

is the so-called non-hermitian mass matrix. Its hermitian parts $M = M^\dagger$ and $\Gamma = \Gamma^\dagger$ are given by

$$M \;=\; \frac{1}{2}(\mathcal{M} + \mathcal{M}^\dagger)\,, \tag{A.25}$$

$$\Gamma \;=\; i(\mathcal{M} - \mathcal{M}^\dagger)\,, \tag{A.26}$$

and we obtain from (A.21)

$$M_{\alpha\alpha'} \;=\; E_0\delta_{\alpha\alpha'} + \langle\alpha|\mathcal{H}'|\alpha'\rangle + \mathcal{P}\sum_{\beta}\frac{\langle\alpha|\mathcal{H}'|\beta\rangle\langle\beta|\mathcal{H}'|\alpha'\rangle}{(E_0 - E_\beta)}\,, \tag{A.27}$$

$$\Gamma_{\alpha\alpha'} \;=\; 2\pi\sum_{\beta}\delta(E_0 - E_\beta)\langle\alpha|\mathcal{H}'|\beta\rangle\langle\beta|\mathcal{H}'|\alpha'\rangle\,. \tag{A.28}$$

A.2 Application to the system of the neutral kaons

In the application of the formalism to the system of the neutral kaons it is important to be aware of the assumptions we have made in the derivation of formula (A.27) and (A.28). Furthermore we have to identify the above notation.

Considering strong and non-leptonic weak interaction in the decay of the neutral kaons, we can take the states $|K^0\rangle$ and $|\overline{K}^0\rangle$ as the non-disturbed eigenstates $|\alpha\rangle$ of the strong Hamiltonian $\mathcal{H}_0$. The small perturbation $\mathcal{H}'$ is the Hamiltonian of the non-leptonic weak interaction, which forces the kaons to decay in the continuous states $|\beta\rangle = |\pi\pi\rangle, |\pi\pi\pi\rangle, \ldots$. Therefore the assumptions made in the Wigner-Weisskopf formalism are first that these states do not decay by virtue of the weak interaction, and second that the weak interaction is small compared to the strong one. Both assumption are naturally fullfilled in the system of the neutral kaons.

The mass matrix $\mathcal{M}$ is then a 2×2-matrix with the eigenvalues $\lambda_{L,S} = M_{L,S} - \frac{i}{2}\Gamma_{L,S}$ and the eigenstates are given by

$$|K_L\rangle = \frac{1}{\sqrt{1+|\tilde{\eta}_L|^2}} \left(|K^0\rangle + \tilde{\eta}_L|\overline{K}^0\rangle\right) \tag{A.29}$$

$$|K_S\rangle = \frac{1}{\sqrt{1+|\tilde{\eta}_S|^2}} \left(|K^0\rangle - \tilde{\eta}_S|\overline{K}^0\rangle\right) . \tag{A.30}$$

CPT-invariance of $\mathcal{H}$ now implies that

$$\langle K^0|\mathcal{M}|K^0\rangle = \langle \overline{K}^0|\mathcal{M}|\overline{K}^0\rangle \, , \tag{A.31}$$

and thus $\tilde{\eta}_L = \tilde{\eta}_S$. Furthermore we can derive from the CPT-invariance of $\mathcal{H}$ the well known relations for the masses and decay width of particle and antiparticle, in this case K^0 and $\overline{K}^0$:

$$\langle K^0|M|K^0\rangle = \langle \overline{K}^0|M|\overline{K}^0\rangle \, , \tag{A.32}$$

$$\langle K^0|\Gamma|K^0\rangle = \langle \overline{K}^0|\Gamma|\overline{K}^0\rangle \, . \tag{A.33}$$

Appendix B

Calculations and results for $\mathcal{L}_{W1}$

B.1 Calculation of the $K \to 2\pi$ amplitudes for $\mathcal{L}_{W1}$

In this section we will calculate the amplitudes of the kaons decaying into two pions, $K \to 2\pi$, for the Lagrangian $\mathcal{L}_{W1}$. We are especially interested in terms where exactly one neutral kaon and two pions are involved, since these contribute to the amplitudes. After expanding U in powers of the fields up to third order and taking only terms of the form $K\pi\pi$ we obtain

$$\mathcal{L}_{W1}(K\pi\pi) = \sqrt{2}F_\pi \left\{ \left(\gamma K^0 + \overline{\gamma}\overline{K}^0\right) \overleftrightarrow{\partial}_\mu \pi^0 \frac{1}{2}\partial^\mu \pi^0 + \partial_\mu \left(\delta K^0 - \overline{\delta}\overline{K}^0\right) \pi^+ \overleftrightarrow{\partial}^\mu \pi^- \right.$$
$$\left. + \omega K^0 \overleftrightarrow{\partial}_\mu \pi^+ \partial^\mu \pi^- + \overline{\omega}\overline{K}^0 \overleftrightarrow{\partial}_\mu \pi^- \partial^\mu \pi^+ \right\} , \quad \text{(B.1)}$$

where γ, δ and ω are set according to table B.1.

Neglecting the strong final state interactions of the pions for the moment, the matrix-element $\mathcal{A}(K^0 \to \pi^+\pi^-)$ yields

$$\begin{aligned} \mathcal{A}(K^0 \to \pi^+\pi^-) &= \langle \pi^+(\vec{k_1})\pi^-(\vec{k_2})|\mathcal{L}_{W1}(0)|K^0(\vec{p})\rangle \\ &= \frac{2}{\sqrt{2}}F_\pi \left(\delta(p\cdot k_1 - p\cdot k_2) - \omega(p\cdot k_1 + k_1\cdot k_2)\right) . \end{aligned} \quad \text{(B.2)}$$

Considering momentum and energy conservation we may compute the Lorentz invariant products $k_1 \cdot k_2, p \cdot k_1$ and $p \cdot k_2$ in the rest frame of the decaying particle. Thus we have the following kinematic relations

$$k_1 \cdot k_2 = \frac{1}{2}M_K^2 - M_\pi^2 , \qquad p\cdot k_1 = p \cdot k_2 = \frac{1}{2}M_K^2 , \quad \text{(B.3)}$$

which lead to the amplitude

$$\mathcal{A}(K^0 \to \pi^+\pi^-) = -\sqrt{2}F_\pi\omega \left(M_K^2 - M_\pi^2\right) . \quad \text{(B.4)}$$

The calculation of the $\mathcal{A}(K^0 \to \pi^0\pi^0)$-amplitude is straightforward and yields

$$\mathcal{A}(K^0 \to \pi^0\pi^0) = \sqrt{2}F_\pi\gamma \left(M_K^2 - M_\pi^2\right) , \quad \text{(B.5)}$$

where we used (B.3) again.

With the Clebsch-Gordan decomposition of the isospin $I = 0$ and $I = 2$ states,

$$|\pi\pi, I=0\rangle = \frac{1}{\sqrt{3}}\left(|\pi^+(\vec{k})\pi^-(-\vec{k})\rangle - |\pi^0(\vec{k})\pi^0(-\vec{k})\rangle + |\pi^+(-\vec{k})\pi^-(\vec{k})\rangle\right), \tag{B.6}$$

$$|\pi\pi, I=2\rangle = \frac{1}{\sqrt{6}}\left(|\pi^+(\vec{k})\pi^-(-\vec{k})\rangle + 2|\pi^0(\vec{k})\pi^0(-\vec{k})\rangle + |\pi^+(-\vec{k})\pi^-(\vec{k})\rangle\right), \tag{B.7}$$

and taking in account now the final state phase shift, δ_0 and δ_2, we get

$$\langle\pi\pi, I=0|\mathcal{L}_{W1}(0)|K^0\rangle = -\sqrt{\frac{2}{3}}F_\pi(2\omega+\gamma)\left(M_K^2 - M_\pi^2\right)e^{i\delta_0} \doteq iA_0 e^{i\delta_0}, \tag{B.8}$$

$$\langle\pi\pi, I=2|\mathcal{L}_{W1}(0)|K^0\rangle = -\frac{2}{\sqrt{3}}F_\pi(\omega-\gamma)\left(M_K^2 - M_\pi^2\right)e^{i\delta_2} \doteq iA_2 e^{i\delta_2}. \tag{B.9}$$

Thus we finally have

$$A_0 = \sqrt{\frac{2}{3}}iF_\pi(2\omega+\gamma)\left(M_K^2 - M_\pi^2\right), \tag{B.10}$$

$$A_2 = \frac{2}{\sqrt{3}}iF_\pi(\omega-\gamma)\left(M_K^2 - M_\pi^2\right), \tag{B.11}$$

where γ and ω can be put according to table B.1.

B.2 Some contributions to the kaon self energy

In this appendix we will present some contributions to the self energy of the kaon calculated from the Lagrangian $\mathcal{L}_{W1}$.

We will consider four different contributions to $\Sigma(p^2)$ (see figure B.1). The Lagrangian $\tilde{\mathcal{L}}_{W1}$ inducing these contributions is obtained by expanding U in powers of the fields up to third order and neglecting all terms where K^+, K^- and η's are involved. We recognize the following general structure of the Lagrangian:

$$\begin{aligned}\tilde{\mathcal{L}}_{W1} = \sqrt{2}F_\pi\Big\{ & iF_\pi\partial_\mu\left(\alpha K^0 - \overline{\alpha}\overline{K}^0\right)\partial^\mu\pi^0 \\ & + \partial_\mu\left(\beta K^0 - \overline{\beta}\,\overline{K}^0\right)\overline{K}^0 \overleftrightarrow{\partial^\mu} K^0 \\ & + \left(\gamma K^0 + \overline{\gamma}\overline{K}^0\right)\overleftrightarrow{\partial_\mu}\pi^0\frac{1}{2}\partial^\mu\pi^0 \\ & + \partial_\mu\left(\delta K^0 - \overline{\delta}\overline{K}^0\right)\pi^+\overleftrightarrow{\partial^\mu}\pi^- \\ & + \omega K^0\overleftrightarrow{\partial_\mu}\pi^+\partial^\mu\pi^- + \overline{\omega}\overline{K}^0\overleftrightarrow{\partial_\mu}\pi^-\partial^\mu\pi^+\Big\}.\end{aligned} \tag{B.12}$$

The octet-, 27-plet- and ($\Delta I = \frac{1}{2}$, $\frac{3}{2}$)-part of $\tilde{\mathcal{L}}_{W1}$ can be obtained by setting the couplings according to Table B.1. Note that we can rewrite the three last terms in $\mathcal{L}_{W1}$ to get

$$+\partial_\mu\left(\delta' K^0 - \overline{\delta'}\,\overline{K}^0\right)\pi^+\overleftrightarrow{\partial^\mu}\pi^- + \omega' K^0\overleftrightarrow{\partial_\mu}\pi^-\partial^\mu\pi^+ + \overline{\omega'}\,\overline{K}^0\overleftrightarrow{\partial_\mu}\pi^+\partial^\mu\pi^- \tag{B.13}$$

with $\delta' = \delta - \omega$ and $\omega' = \omega$.

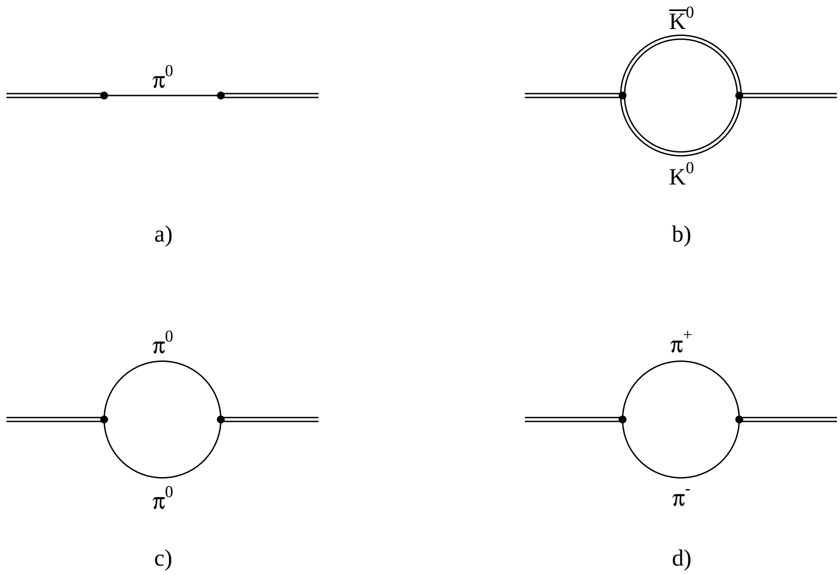

Figure B.1: Contributions of a) a pion propagator, b) a neutral kaon loop c) a neutral pion loop and d) a charged pion loop to the self energy of the kaon. A simple line represents a pion, whereas a double line denotes a kaon.

Table B.1: Couplings $c_i = c_i^- - ic_i^+$ appearing in the different parts of $\tilde{\mathcal{L}}_{W1}$. Note that c_3, c_3' and c_3'' are related by $c_3' = \frac{1}{3}c_3$ and $c_3'' = \frac{5}{3}c_3$, respectively.

	α	β	γ	δ	ω
$\tilde{\mathcal{L}}_{W1}^{8}$	$-c_2$	0	$-c_2$	$-c_2$	$-c_2$
$\tilde{\mathcal{L}}_{W1}^{27}$	$3c_3$	0	$3c_3$	$3c_3$	$-2c_3$
$\tilde{\mathcal{L}}_{W1}^{27}(\Delta I = 1/2)$	$-c_3'$	$-5c_3'$	$-c_3'$	$-c_3'$	$-c_3'$
$\tilde{\mathcal{L}}_{W1}^{27}(\Delta I = 3/2)$	$2c_3''$	c_3''	$2c_3''$	$2c_3''$	$-c_3''$
$\tilde{\mathcal{L}}_{W1}$	$(3c_3 - c_2)$	0	$(3c_3 - c_2)$	$(3c_3 - c_2)$	$-(2c_3 + c_2)$

First, there is a contribution coming from a term in the Lagrangian which is quadratic in the fields, i.e. the first line in (B.12). Then the terms cubic in the fields yield two kind of pion loops, neutral and charged ones, and a kaon loop.

We will now present the various contributions a) - d). The indices 1 and 2 stand for K^0 and $\overline{K}^0$, respectively:

$$\Sigma_{11}^{a)}(p^2) = \Sigma_{22}^{a)}(p^2) = 2\alpha\overline{\alpha}F_\pi^4 \cdot K(p^2; M_\pi)\,, \tag{B.14}$$

$$\Sigma_{12}^{a)}(p^2) = \overline{\Sigma_{21}^{a)}(p^2)} = -2\overline{\alpha}^2 F_\pi^4 \cdot K(p^2; M_\pi)\,, \tag{B.15}$$

$$\begin{aligned}\Sigma_{11}^{b)}(p^2) &= \Sigma_{22}^{b)}(p^2) = -6\beta\overline{\beta}F_\pi^2\Big\{\left(p^2 - M_K^2\right)^2 J(p^2; M_K, M_K) \\ &\qquad -2M_K^4 \cdot T(M_K)\Big\}\,,\end{aligned} \tag{B.16}$$

$$\begin{aligned}\Sigma_{12}^{b)}(p^2) &= -2\overline{\beta}^2F_\pi^2\Big\{\left(p^2 - M_K^2\right)^2 J(p^2; M_K, M_K) \\ &\quad + \left(6p^2 - 2M_K^2\right) M_K^2 T(M_K)\Big\}\,,\end{aligned} \tag{B.17}$$

$$\begin{aligned}\Sigma_{21}^{b)}(p^2) &= -2\beta^2F_\pi^2\Big\{\left(p^2 - M_K^2\right)^2 J(p^2; M_K, M_K) \\ &\quad + \left(6p^2 - 2M_K^2\right) M_K^2 \cdot T(M_K)\Big\}\,,\end{aligned} \tag{B.18}$$

$$\begin{aligned}\Sigma_{11}^{c)}(p^2) &= \Sigma_{22}^{c)}(p^2) = -2\gamma\overline{\gamma}F_\pi^2\Big\{\left(p^2 - M_\pi^2\right)^2 J(p^2; M_\pi, M_\pi) \\ &\qquad + \left(\frac{3}{2}p^2 - 2M_\pi^2\right) M_\pi^2 \cdot T(M_\pi)\Big\}\,,\end{aligned} \tag{B.19}$$

$$\begin{aligned}\Sigma_{12}^{c)}(p^2) &= -\overline{\gamma}^2F_\pi^2\Big\{\left(p^2 - M_\pi^2\right)^2 J(p^2; M_\pi, M_\pi) \\ &\quad + \left(\frac{3}{2}p^2 - 2M_\pi^2\right) M_\pi^2 \cdot T(M_\pi)\Big\}\,,\end{aligned} \tag{B.20}$$

$$\begin{aligned}\Sigma_{21}^{c)}(p^2) &= -\gamma^2F_\pi^2\Big\{\left(p^2 - M_\pi^2\right)^2 J(p^2; M_\pi, M_\pi) \\ &\quad + \left(\frac{3}{2}p^2 - 2M_\pi^2\right) M_\pi^2 \cdot T(M_\pi)\Big\}\,,\end{aligned} \tag{B.21}$$

$$\begin{aligned}\Sigma_{11}^{d)}(p^2) &= \Sigma_{22}^{d)}(p^2) = -2F_\pi^2\Big\{\overline{\omega}\omega\left(p^2 - M_\pi^2\right)^2 J(p^2; M_\pi, M_\pi) \\ &\quad + \left((-2\delta\overline{\delta} + \omega\overline{\delta} + \delta\overline{\omega} + \overline{\omega}\omega)p^2 - 2\overline{\omega}\omega M_\pi^2\right) M_\pi^2 \cdot T(M_\pi)\Big\}\,,\end{aligned} \tag{B.22}$$

$$\begin{aligned}\Sigma_{12}^{d)}(p^2) &= -2F_\pi^2\Big\{\overline{\omega}^2\left(p^2 - M_\pi^2\right)^2 J(p^2; M_\pi, M_\pi) \\ &\quad +2\left(\overline{\delta}^2 - \overline{\delta}\overline{\omega} + \overline{\omega}^2)p^2 - \overline{\omega}^2 M_\pi^2\right) M_\pi^2 \cdot T(M_\pi)\Big\}\,,\end{aligned} \tag{B.23}$$

$$\begin{aligned}\Sigma_{21}^{d)}(p^2) &= -2F_\pi^2\Big\{\omega^2\left(p^2 - M_\pi^2\right)^2 J(p^2; M_\pi, M_\pi) \\ &\quad +2\left((\delta^2 - \delta\omega + \omega^2)p^2 - \omega^2 M_\pi^2\right) M_\pi^2 \cdot T(M_\pi)\Big\}\,,\end{aligned} \tag{B.24}$$

where

$$K(p^2;M) = \frac{p^4}{p^2 - M^2}\,, \tag{B.25}$$

and $J(p^2;M_1,M_2)$ and $M^2\cdot T(M)$ are the loop functions

$$\begin{aligned} J(p^2;M_1,M_2) &= \frac{1}{i}\int \frac{d^4l}{(2\pi)^4}\frac{1}{M_1{}^2 - l^2 - i\varepsilon}\cdot\frac{1}{M_2{}^2 - (p-l)^2 - i\varepsilon}\,, & \text{(B.26)}\\ M^2\cdot T(M) &= \frac{1}{i}\int \frac{d^4l}{(2\pi)^4}\frac{1}{M^2 - l^2 - i\varepsilon}\,, & \text{(B.27)} \end{aligned}$$

which have to be renormalised, since they are logarithmically and quadratically divergent, respectively. The necessary counterterms are of order $\mathcal{O}(G_F{}^2)$ and contribute in first order of $\mathcal{L}_{ct}$ to $\Sigma(p^2)$ via $-i\langle K_i|\mathcal{L}_{ct}|K_j\rangle_{1\mathrm{PI}}$ cancelling the divergent parts of it. Instead, we regularize the integrals by working in $d \neq 4$ dimensions in the following.

$M^2\cdot T(M;d)$ is constant in the whole p^2-plane, while $J(p^2;M,M;d)$ is only analytic in the cut p^2-plane, where the cut runs from $p^2 = 4M^2$ along the real positive axis. Since for the computation of the mass matrix we have to take the kaon on the mass shell, i.e. $p^2 = M_K^2 > 4M_\pi^2$, $J(M_K^2;M_\pi,M_\pi;d)$ will have an imaginary part. However, this is not the case for $J(M_K^2;M_K,M_K;d)$, where the cut runs only from $p^2 = 4M_K^2$ along the real positive axis. In particular we have no contribution to the decay width of the kaon from the kaon loop. Furthermore, the contribution from $J(M_K^2;M_K,M_K;d)$ to the mass matrix vanishes anyway, because the factor $(p^2 - M_K^2)^2$ in $\Sigma_{ij}(p^2)$ equals zero if we put the momentum of the kaon on the mass shell.

Collecting now all finite contributions a) - d) of $\tilde{\mathcal{L}}_{W1}$ to the self energy of the kaon we can write $\Sigma_{ij}(p^2)$ in the form $(i,j=1,2)$

$$\begin{aligned} \Sigma_{ij}(p^2) &= -F_\pi^2\Big\{\sigma_{ij}^1 2F_\pi^2 K(p^2;M_\pi{}^2) + \left(\sigma_{ij}^2 p^2 - \sigma_{ij}^3 M_\pi^2\right) M_\pi^2\cdot T(M_\pi;d)\\ &\quad + \left(\sigma_{ij}^4 p^2 - \sigma_{ij}^5 M_K^2\right) M_K^2\cdot T(M_K;d)\\ &\quad +\sigma_{ij}^6\left(p^2 - M_\pi^2\right) J(p^2;M_\pi,M_\pi;d)\\ &\quad +\sigma_{ij}^7\left(p^2 - M_K^2\right)^2 J(p^2;M_K,M_K;d)\Big\}\,. \end{aligned} \tag{B.28}$$

where the coefficients σ_{11}^i, $i=1,\ldots,6$ for $\Sigma_{11}(p^2)$ are given by

$$\begin{aligned} \sigma_{11}^1 &= -\alpha\overline{\alpha}\,,\\ \sigma_{11}^2 &= \frac{3}{2}\gamma\overline{\gamma} + 2(-2\delta\overline{\delta} + \omega\overline{\delta} + \delta\overline{\omega} + \overline{\omega}\omega)\,,\\ \sigma_{11}^3 &= 2\gamma\overline{\gamma} + 4\overline{\omega}\omega\,,\\ \sigma_{11}^4 &= 0\,,\\ \sigma_{11}^5 &= 12\beta\overline{\beta}\,,\\ \sigma_{11}^6 &= \gamma\overline{\gamma} + 2\overline{\omega}\omega\,,\\ \sigma_{11}^7 &= 6\beta\overline{\beta}\,. \end{aligned} \tag{B.29}$$

The coefficients σ_{22}^i are equal to σ_{11}^i since $\Sigma_{11}(p^2) = \Sigma_{22}(p^2)$:

$$\sigma_{22}^i = \sigma_{11}^i\,. \tag{B.30}$$

For the off-diagonal element of the self energy, $\Sigma_{21}(p^2)$, we have

$$\begin{aligned}
\sigma_{21}^1 &= \alpha^2\,, \\
\sigma_{21}^2 &= \frac{3}{2}\gamma^2 + 4(\delta^2 - \delta\omega + \omega^2)\,, \\
\sigma_{21}^3 &= 2\gamma^2 + 4\omega^2\,, \\
\sigma_{21}^4 &= 12\beta^2\,, \\
\sigma_{21}^5 &= 4\beta^2\,, \\
\sigma_{21}^6 &= \gamma^2 + 2\omega^2\,, \\
\sigma_{21}^7 &= 2\beta^2\,.
\end{aligned} \tag{B.31}$$

The coefficients c_{12}^i for $\Sigma_{12}(p^2)$ are related to c_{21}^i by complex conjugation

$$c_{12}^i = \overline{c_{21}^i}\,. \tag{B.32}$$

Appendix C

Conventions

C.1 The pseudo-scalar meson fields

This section will outline the phase conventions for the pseudo-scalar meson fields. The phases of the creation and annihilation operators are chosen following the Condon-Shortley phase convention. They are as follows:

$$\begin{aligned}
\pi^+(x) &= -\int d_\mu(p)\left\{e^{-ipx}\,a_{\pi^+}(p) - e^{ipx}\,a^\dagger_{\pi^-}(p)\right\}\,, \\
\pi^-(x) &= \int d_\mu(p)\left\{e^{-ipx}\,a_{\pi^-}(p) - e^{ipx}\,a^\dagger_{\pi^+}(p)\right\}\,, \\
\pi^0(x) &= \int d_\mu(p)\left\{e^{-ipx}\,a_{\pi^0}(p) + e^{ipx}\,a^\dagger_{\pi^0}(p)\right\}\,, \\
K^0(x) &= -\int d_\mu(p)\left\{e^{-ipx}\,a_{K^0}(p) - e^{ipx}a^\dagger_{\overline{K}^0}(p)\right\}\,, \\
\overline{K}^0(x) &= \int d_\mu(p)\left\{e^{-ipx}\,a_{\overline{K}^0}(p) - e^{ipx}\,a^\dagger_{K^0}(p)\right\}\,,
\end{aligned} \tag{C.1}$$

where $d_\mu(p)$ is the covariant integration measure $\frac{d^3p}{(2\pi)^3}\frac{1}{2p_0}$. With these conventions we can derive the following expectation values of the meson fields:

$$\begin{aligned}
\langle 0|\pi^+(0)|\pi^+\rangle &= -1\,, \\
\langle 0|\pi^-(0)|\pi^-\rangle &= +1\,, \\
\langle 0|\pi^0(0)|\pi^0\rangle &= +1\,, \\
\langle 0|K^0(0)|K^0\rangle &= -1\,, \\
\langle 0|\overline{K}^0(0)|\overline{K}^0\rangle &= -1\,.
\end{aligned} \tag{C.2}$$

The field matrix of the octet of pseudoscalar fields is such that

$$\phi = \sum_a^8 \lambda_a \phi^a = \sqrt{2}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \overline{K}^0 & -2\frac{\eta}{\sqrt{6}} \end{pmatrix}\,. \tag{C.3}$$

C.2 The isospin amplitudes A_I

This section is designated to give a compilation of the different conventions used in the literature to define the isospin amplitudes A_I. Although not complete it is intended to prevent confusion. In addition we give the phase convention of the CP-transformation used by the authors.

The Particle Data Group [4] uses $CP|K^0\rangle = +|\overline{K}^0\rangle$ and defines

$$\langle I|T|K^0\rangle = A_I e^{i\delta_I}\,, \tag{C.4}$$

while T is not defined.

Nachtmann [6] uses $CP|K^0\rangle = -|\overline{K}^0\rangle$ and defines

$$\langle \pi\pi, I|T|K^0\rangle = A_I e^{i\delta_I}\,, \tag{C.5}$$

where T is defined through the scattering matrix $S = 1 + i(2\pi)^4\delta(P)\cdot T$.

Grimus [7] uses $CP|K^0\rangle = -|\overline{K}^0\rangle$ and defines

$$\langle \pi\pi, I out| - i\mathcal{H}_{eff}(0)|K^0\rangle = A_I e^{i\delta_I}\,, \tag{C.6}$$

where $\mathcal{H}_{eff}(0)$ is the effective weak Hamiltonian density at $x = 0$.

De Rafael [8] uses $CP|K^0\rangle = -|\overline{K}^0\rangle$ and defines

$$\langle I|T|K^0\rangle = iA_I e^{i\delta_I}\,, \tag{C.7}$$

where $S = 1 + iT$. We follow this convention upon a factor $(2\pi)^4\delta(P)$ throughout this work.

Wu and Yang [10] use $CP|K^0\rangle = +|\overline{K}^0\rangle$ following Lee, Oehme and Yang [15] and denote the decay amplitude $K^0 \to \pi\pi$, I standing wave as A_I.

Maiani [16] uses $CP|K^0\rangle = +|\overline{K}^0\rangle$ and defines

$$\langle 2\pi, I; out|H_W|K^0\rangle = \sqrt{\frac{3}{2}}A_I e^{i\delta_I}\,, \tag{C.8}$$

where H_W is the weak Hamiltonian.

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